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PHYSICS

EVALUATION PLAN

1. (a) Theory/Written examination (3 hours) : **70 marks**
   (b) Practical examination (3 hours) : **30 marks**
   Total : **100 marks**

2. Question paper pattern for the theory/written examination :

Section	Question Type	Question No.	Internal Choice	Total Marks	Marks with Option
A	Multiple Choice Questions (MCQ)	Q. 1. [(i) to (x)]	–	10	10
	Very Short Answer Questions (VSA)	Q. 2. [(i) to (viii)]	–	8	8
B	Short Answer Questions (SA) – I	Q. 3. to Q. 14.	8 out of 12 Qs.	16	24
C	Short Answer Questions (SA) – II	Q. 15. to Q. 26.	8 out of 12 Qs.	24	36
D	Long Answer Questions (LA)	Q. 27. to Q. 31.	3 out of 5 Qs.	12	20
				70	**98**

3. Chapterwise distribution of marks in the question paper :

Chapter No.	Name of the Chapter	Marks	Marks with Option
1	Rotational Dynamics	5	7
2	Mechanical Properties of Fluids	5	7
3	Kinetic Theory of Gases and Radiation	5	7
4	Thermodynamics	5	7
5	Oscillations	4	5
6	Superposition of Waves	4	6
7	Wave Optics	5	7
8	Electrostatics	4	6
9	Current Electricity	4	6
10	Magnetic Fields due to Electric Current	4	6
11	Magnetic Materials	4	5
12	Electromagnetic Induction	5	7
13	AC Circuits	4	6
14	Dual Nature of Radiation and Matter	4	5
15	Structure of Atoms and Nuclei	4	6
16	Semiconductor Devices	4	5
	Total	**70**	**98**

NON-EVALUATIVE PORTION FOR THE ACADEMIC YEAR 2020-21 AS DECLARED ON 22-07-2020

Chapter No. & Name	Non-evaluative portion
1. Rotational Dynamics	1.4.2 : Sphere of Death
	1.4.3 : Vehicle at the Top of a Convex Over Bridge
	1.11 : Rolling Motion
2. Mechanical Properties of Fluids	2.3 : Pressure
	2.8 : Equation of Continuity
	2.9 : Bernoulli Equation
3. Kinetic Theory of Gases and Radiation	3.2 : Behaviour of a Gas
	3.3 : Ideal Gas and Real Gas
	3.4 : Mean Free Path
	3.8 : Law of Equipartition of Energy
4. Thermodynamics	4.8 : Heat Engines
	4.9 : Refrigerators and Heat Pumps
	4.10 : Second Law of Thermodynamics
	4.11 : Carnot Cycle and Carnot Engine
	4.12 : Sterling Cycle
5. Oscillations	5.7 : Reference Circle Method
	5.9 : Graphical Representation of S.H.M.
	5.14 : Damped Oscillations
	5.15 : Free Oscillations, Forced Oscillations and Resonance
6. Superposition of Waves	6.3 : Reflection of Waves
	6.10 : Characteristics of Sound
	6.11 : Musical Instruments
7. Wave Optics	7.2.1 : Corpuscular Nature
	7.6 : Refraction of a Light at a Plane Boundary Between Two Media
	7.7 : Polarization
	7.10 : Resolving Power
8. Electrostatics	8.5 : Equipotential Surfaces
	8.7 : Conductors and Insulators, Free Charges and Bound Charges
	8.11 : Displacement Current
	8.13 : Van de Graaff Generator

10. Magnetic Fields due to Electric Current	10.3 : Cyclotron Motion
	10.4 : Helical Motion
11. Magnetic Materials	11.2 : Torque Acting on a Magnetic Dipole in a Uniform Magnetic Field
	11.5 : Magnetic Properties of Materials
	11.6 : Hysteresis
	11.7 : Permanent Magnet and Electromagnet
	11.8 : Magnetic Shielding
12. Electromagnetic Induction	12.6 : Induced emf in a Stationary Coil in a Changing Magnetic Field
	12.7 : Generator
	12.8 : Back emf and Back Torque
	12.13 : Energy Density of a Magnetic Field
13. AC Circuits	13.2 : AC Generator
	13.6 : Power in AC Circuits
	13.9 : Sharpness of Resonance : Q Factor
	13.10 : Choke Coil
14. Dual Nature of Radiation and Matter	Table 14.2 : Summary of Analysis of Observations from Experiments on Photoelectric Effect
	14.4 : Photo Cell
	14.6 : Davisson and Germer Experiment
15. Structure of Atoms and Nuclei	15.3 : Geiger Marsden Experiment
	15.7 : Atomic Nucleus
	15.8 : Nuclear Binding Energy
	15.9 : Radioactive Decays
	15.11 : Nuclear Energy
16. Semiconductor Devices	16.3.1 : Zener Diode

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PART 1

MODEL QUESTION PAPER
(WITH SOLUTION AND MARKING SCHEME)

PHYSICS

Time : 3 Hours] **[Max. Marks : 70**

General Instructions :

1. *The question paper is divided into four sections :*
   - *(1)* ***Section A :*** *Q. No. 1 contains* ***10 multiple choice*** *type questions carrying* ***one mark*** *each.*
     Q. No. 2 contains ***8 very short answer*** *type questions carrying* ***one mark*** *each.*
   - *(2)* ***Section B :*** *Q. No. 3 to Q. No. 14 are* ***12 short answer–I*** *type questions carrying* ***two marks*** *each. Attempt any* ***eight*** *questions.*
   - *(3)* ***Section C :*** *Q. No. 15 to Q. No. 26 are* ***12 short answer–II*** *type questions carrying* ***three marks*** *each. Attempt any* ***eight*** *questions.*
   - *(4)* ***Section D :*** *Q. No. 27 to Q. No. 31 are* ***5 long answer*** *type questions carrying* ***four marks*** *each. Attempt any* ***three*** *questions.*
2. *Start each section on a new page.*
3. *Figures to the right indicate full marks.*
4. *For each MCQ, correct answer must be written along with its alphabet :*
   e.g., (a) ...... / (b) ...... / (c) ...... / (d) ....
5. *Evaluation of each MCQ would be done for the first attempt only.*
6. *Use of Logarithm Tables is allowed. Use of a calculator is* ***not*** *allowed.*

Physical constants :

(1) $\pi = 3.142$ (2) $g = 10\ \text{m/s}^2$ (3) $h = 6.63 \times 10^{-34}\ \text{J·s}$ (4) $c = 3 \times 10^8\ \text{m/s}$
(5) $e = 1.6 \times 10^{-19}\ \text{C}$ (6) $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N·m}^2$ (7) $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\text{·K}^4$
(8) $m_e = 9.1 \times 10^{-31}\ \text{kg}$

SECTION – A

Q. 1. Select and write the correct answers to the following questions : [10]

(i) The adiabatic constant for polyatomic molecules having f vibrational modes is

(a) $\frac{f+5}{f+3}$ (b) $\frac{f+3}{f+4}$ (c) $\frac{f+9}{f+7}$ (d) $\frac{f+4}{f+3}$. (1)

(ii) The equation $E = pc$ is valid for

(a) all sub-atomic particles (b) an electron but not for a photon
(c) a photon but not for an electron (d) both an electron and a photon. (1)

(iii) A body of mass 2 kg performs linear SHM. The restoring force acting on it is 3 N when it is 0.06 m from the mean position.

The differential equation of its motion is

(a) $\frac{d^2x}{dt^2} + 100x = 0$ (b) $\frac{d^2x}{dt^2} + 25x = 0$

(c) $50\frac{d^2x}{dt^2} + x = 0$ (d) $2\frac{d^2x}{dt^2} + 3x = 0$. (1)

(iv) An ideal inductor is connected to an ac source. The driving voltage is $V = V_0 \sin \omega t$. The current through the inductor is

(a) zero (b) $\frac{V_0 \sin(\omega t - 90°)}{\omega L}$

(c) $\frac{V_0 \sin(\omega t + 90°)}{\omega L}$ (d) $\frac{V_0 \sin(\omega t + 180°)}{\omega L}$. (1)

(v) When two waves superimpose at a point, the amplitude of the resultant wave depends upon

(a) the amplitude of each wave

(b) the phase difference between the waves

(c) both (a) and (b)

(d) neither (a) nor (b). (1)

(vi) What is the energy required to build up a current of 1 A in an inductor of 20 mH?

(a) 10 mJ (b) 20 mJ (c) 20 J (d) 10 J (1)

(vii) As wavefronts pass normally from one medium to another in which the speed of propagation is altered, the wavelength

(a) remains unchanged (b) increases

(c) decreases (d) may increase or decrease. (1)

(viii) If μ_0 is the permeability of free space and χ_m is the magnetic susceptibility of a medium, the relative permeability of the medium is

(a) $1 + \chi_m$ (b) $\frac{\mu_0}{1 + \chi_m}$ (c) $\frac{\mu_0}{\chi_m}$ (d) $\mu_0(1 + \chi_m)$. (1)

(ix) A parallel plate capacitor is charged and then isolated. On decreasing the plate separation, the potential difference and capacitance respectively

(a) increases, increases (b) increases, decreases

(c) decreases, increases (d) decreases, decreases. (1)

(x) The energy stored in a soap bubble of diameter 6 cm and surface tension 0.04 N/m, is nearly

(a) 0.9×10^{-3} J (b) 0.4×10^{-3} J

(c) 0.7×10^{-3} J (d) 0.5×10^{-3} J. (1)

Q. 2. Answer the following questions : **[8]**

(i) What is the resonance condition in a cyclotron? (1)

(ii) Obtain the dimensions of surface tension. (1)

(iii) Find the maximum speed with which a car can be safely driven along a curve of radius 100 m, if the coefficient of friction between its tyres and the road is 0.2. (1)

(iv) What is the effect of the intensity of incident radiation on the stopping potential in photoelectric emission? (1)

(v) State Wien's displacement law. (1)

(vi) The cross-sectional area of a bar magnet 10 cm long is 1.2 cm^2. If the magnetic moment of the magnet is 2.4 $A \cdot m^2$, find its magnetization. (1)

(vii) On what factors does the potential gradient of a potentiometer wire depend? (1)

(viii) What is nuclear fusion? (1)

SECTION – B

Attempt *any eight* of the following questions : **[16]**

Q. 3. Starting from rest, a body rolls down along an incline that rises by 2 in every 3 along the plane. The body attains a speed of $2\sqrt{3}$ m/s as it travels a distance of $\frac{3}{2}$ m along the incline. What could be the possible shape(s) of the body ? (2)

Q. 4. A solar cooker and a pressure cooker both are used to cook food. Treating them as thermodynamic systems, discuss the similarities and differences between them. (2)

Q. 5. Draw a neat labelled schematic diagram of the structure of a planar photodiode. (2)

Q. 6. The amplitude of a wave is represented by $y = 0.2 \sin 4\pi \left[\frac{t}{0.08} - \frac{x}{0.8}\right]$ in SI units. Find the (a) wavelength (b) frequency and (c) amplitude of the wave. (2)

Q. 7. State the drawbacks of Rutherford's atomic model. (2)

Q. 8. If the difference in speeds of light in glass and water is 2.505×10^7 m/s, find the speed of light in air. [Refractive index of glass = 1.5, refractive index of water = 1.333] (2)

Q. 9. A resistor with resistance R carries a sinusoidally varying AC. Obtain an expression for the heat produced in the resistor in one complete cycle. (2)

Q. 10. Explain why the magnetic force on a charged particle cannot change the linear speed and the kinetic energy of the particle. (2)

Q. 11. A 2000 turns search coil, each of area 1.5 cm^2, is rapidly moved out of a magnetic field of 0.60 T in 0.3 s. Calculate the emf induced in the search coil. (2)

Q. 12. Draw a neat labelled diagram of a Carnot cycle. (2)

Q. 13. State the factors on which the total energy of a particle executing SHM depends. (2)

Q. 14. Three capacitors of capacities 8 μF, 8 μF and 4 μF are connected in series and a potential difference of 120 V is maintained across the combination. Calculate the charge on the 4 μF capacitor. (2)

SECTION – C

Attempt *any eight* of the following questions : **[24]**

Q. 15. A sample of fossilized wood has a carbon-14 decay rate of 6.00 disintegrations per minute per gram. Detemine the age of the sample. The decay rate of carbon-14 in fresh wood today is 13.6 counts per minute per gram and the decay constant of carbon-14 is 3.84×10^{-12} per second. (3)

Q. 16. Derive an expression for capillary rise for a liquid having a concave meniscus. (3)

Q. 17. Define an AND gate. Give the logic symbol, Boolean expression and truth table for an AND gate. (3)

Q. 18. A small blackened solid copper sphere of radius 2.5 cm and absorption coefficient 0.9 is placed in an evacuated chamber. The temperature of the chamber is maintained at 27 °C. At what rate must energy be supplied to the copper sphere to maintain its temperature at 127 °C ? (3)

Q. 19. A device Y is connected across an AC source of emf $e = e_0 \sin \omega t$. The current through Y is given as $i = i_0 \sin (\omega t + \pi/2)$.

(a) Identify the device Y and write the expression for its reactance. (1)

(b) Draw graphs showing variation of emf and current with time over one cycle of AC for Y. (1)

(c) Draw the phasor diagram for the device Y. (1)

Q. 20. An ideal monatomic gas is adiabatically compressed so that its final temperature is twice its initial temperature. What is the ratio of the final pressure to its initial pressure ? (3)

Q. 21. Discuss analytically the formation of beats and show that the beat frequency equals the difference in frequencies of two interfering waves. (3)

Q. 22. Obtain an expression for the electric field intensity at a point outside an infinitely long charged cylindrical conductor. (3)

Q. 23. Calculate the momentum of an electron and the associated wavelength when its kinetic energy is 150 eV. (3)

Q. 24. A magnetic dipole of moment 0.025 J/T is free to rotate in a uniform magnetic field of induction 50 mT. When released from rest in the magnetic field, the dipole rotates to align with the field. At the instant the dipole moment is parallel to the field, its kinetic energy is 625 μJ. What was the initial angle between the dipole moment and the magnetic field ? (3)

Q. 25. Explain with a neat circuit diagram, how you will determine the unknown resistance using a metre bridge. (3)

Q. 26. Explain the origin of paramagnetism on the basis of atomic structure. (3)

SECTION – D

Attempt *any three* of the following questions : **[12]**

Q. 27. (a) Prove that the average kinetic energy per molecule of an ideal gas is $\frac{3}{2} k_B T$. (2)

(b) In a hydraulic lift, the input piston has surface area 25 cm^2. The output piston has surface area 1000 cm^2. If a force of 50 N is applied to the input piston, it raises the output piston by 2 m. Calculate the weight on the output piston. (2)

Q. 28. State and prove the theorem of parallel axis. (4)

Q. 29. (a) Explain the Rayleigh criterion for the limit of resolution for two linear objects. (2)

(b) If a glass plate of refractive index 1.732 is to be used as a polarizer, what would be the (i) polarizing angle and (ii) angle of refraction ? (2)

Q. 30. State the principle of working of a transformer. Describe the construction of a transformer. Derive the relationship $\frac{V_P}{V_S} = \frac{I_S}{I_P}$ for a transformer. (4)

Q. 31 (a) A particle executing SHM has velocities v_1 and v_2 when at distances x_1 and x_2 respectively from the mean position. Show that its period is $T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$ and the amplitude of SHM is $A = \sqrt{\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}$. (2)

(b) What will a voltmeter of resistance 250 Ω read when it is connected across a cell of emf 2 V and internal resistance 12 Ω ? (2)

SECTION - A

Note : Q. 1 is a set of 10 multiple choice type questions. Do not rewrite the statements. Write the question number, the option number [viz., (a), (b), (c) or (d)] as well as the option in full. Calculation, if required, may be done at the bottom of the page so that you may recheck later. Answer Q. 2 very briefly.

Q. 1. (i) (d) $\frac{f+4}{f+3}$ (1 mark)

(ii) (c) a photon but not for an electron (1 mark)

(iii) (b) $\frac{d^2x}{dt^2} + 25x = 0$ (1 mark)

(iv) (b) $\frac{V_0 \sin(\omega t - 90°)}{\omega L}$ (1 mark)

(v) (c) both (a) and (b) (1 mark)

(vi) (a) 10 mJ (1 mark)

(vii) (d) may increase or decrease (1 mark)

(viii) (d) $\mu_0(1+\chi_m)$ (1 mark)

(ix) (a) increases, increases (1 mark)

(x) (a) 0.9×10^{-3} J (1 mark)

Solution (rough work)

(iii) $k = \frac{F}{x} = \frac{3N}{6 \times 10^{-2} m} = \frac{100}{2} = 50$ N/m.

$\omega^2 = \frac{k}{m} = \frac{50 \text{ N/m}}{2 \text{ kg}} = 25 \text{ s}^{-2}$, so that the differential equation

of SHM, $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$, is $\frac{d^2x}{dt^2} + 25x = 0$,

(vi) $W = \frac{1}{2}LI^2 = \frac{1}{2}(20 \times 10^{-3})(1)^2 = 10 \times 10^{-3}$ J

(x) $W = 8\pi r^2 T = 8 \times 3.142 \times (3 \times 10^{-2})^2 \times 0.04$

$\simeq 32 \times 9 \times 3.142 \times 10^{-6} \simeq 32 \times 28 \times 10^{-6}$

$= 896 \times 10^{-6} \simeq 0.9 \times 10^{-3}$J

Q. 2. (i) The frequency of the alternating voltage between the dees of a cyclotron should be equal to the cyclotron frequency so that a positive ion exiting a dee always sees an accelerating potential difference to the other dee. This equality of the frequencies is called the resonance condition. (1 mark)

(ii) Surface tension is a force per unit length.

$$\therefore \text{[Surface tension]} = \frac{\text{[force]}}{\text{[length]}} = \frac{[ML^1T^{-2}]}{[M^0L^1T^0]} = [ML^0T^{-2}]$$

(1 mark)

(iii) The maximum speed,

$v = \sqrt{r\mu_s g} = \sqrt{100 \times 0.2 \times 9.8} = \sqrt{196} = 14$ m/s (1 mark)

(iv) For a given frequency of incident radiation above the threshold, the stopping potential is independent of the intensity of radiation.

(1 mark)

(v) Wien's displacement law : The wavelength for which the emissive power of a blackbody is maximum, is inversely proportional to the absolute temperature of the blackbody. (1 mark)

(vi) Magnetization,

$$M_Z = \frac{M}{V} = \frac{M}{LA} = \frac{2.4}{(0.1)(1.2 \times 10^{-4})} = 2 \times 10^5 \text{ A/m}$$

(1 mark)

(vii) The potential gradient along a potentiometer wire depends upon the potential difference between the ends of the wire and the length of the wire. (1 mark)

(viii) A type of nuclear reaction in which lighter atomic nuclei (of low atomic number) fuse to form a heavier nucleus (of higher atomic number) with the release of enormous amount of energy is called nuclear fusion.

(1 mark)

SECTION - B

Q. 3. Data : $u = 0$, $\sin\theta = \frac{2}{3}$, $v = 2\sqrt{3}$ m/s, $L = \frac{3}{2}$ m, $g = 10$ m/s^2

$$v = \sqrt{\frac{2gL\sin\theta}{1 + (k^2/R^2)}} = \sqrt{\frac{2gL\sin\theta}{1+\beta}}$$ (½ mark)

where k is the radius of gyration and R is the radius of the body of circular or spherical symmetry.

$$\therefore\ v^2 = \frac{2gL\sin\theta}{1+\beta}$$

$$\therefore\ 1+\beta = \frac{2gL\sin\theta}{v^2}$$

$$= \frac{2\,(10\ \text{m/s}^2)\left(\frac{3}{2}\text{ m}\right)\left(\frac{2}{3}\right)}{(2\sqrt{3}\text{ m/s})^2}$$ (½ mark)

$$= \frac{20}{12} = \frac{5}{3}$$

$$\therefore\ \beta = \frac{k^2}{R^2} = \frac{5}{3} - 1 = \frac{2}{3}$$ (½ mark)

Therefore, the body rolling down is a hollow sphere. (½ mark)

Q. 4. Similarities :

(i) Heat is added to the system.

(ii) There is increase in the internal energy of the system.

(iii) Work is done by the system on its environment. (1 mark)

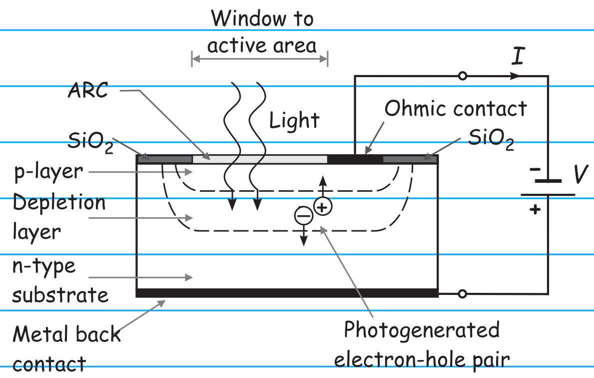
Differences :

In a solar cooker, heat is supplied in the form of solar radiation. The rate of supply of heat is relatively low.

In a pressure cooker, usually LPG is used (burned) to provide heat. The rate of supply of heat is relatively high.

As a result, it takes very long time for cooking when a solar cooker is used. With a pressure cooker, it does not take very long time for cooking. (1 mark)

Q. 5.



V : Reverse bias supply, I : Photocurrent,
ARC : Antireflective silicon nitride coating

Schematic structure of a planar photodiode.

(Diagram : 1 mark, Labelling : 1 mark)

Q. 6. Data : $y = 0.2 \sin 4\pi \left[\frac{t}{0.08} - \frac{x}{0.8}\right] = 0.2 \sin 2\pi \left[\frac{t}{0.04} - \frac{x}{0.4}\right]$

Let us compare the above equation with the equation of a simple harmonic progressive wave :

$y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right] = 0.2 \sin 2\pi \left[\frac{t}{0.04} - \frac{x}{0.4}\right]$ ($\frac{1}{2}$ mark)

Comparing the quantities on both sides, we get,

$A = 0.2$ m, $T = 0.04$ s, $\lambda = 0.4$ m

Therefore,

(a) Wavelength $(\lambda) = 0.4$ m ($\frac{1}{2}$ mark)

(b) Frequency $(n) = \frac{1}{T} = \frac{1}{0.04} = 25$ Hz ($\frac{1}{2}$ mark)

(c) Amplitude $(A) = 0.2$ m ($\frac{1}{2}$ mark)

Q. 7. (1) According to Rutherford, the electrons revolve in circular orbits around the atomic nucleus. The circular motion is an accelerated motion. According to the classical electromagnetic theory, an accelerated charge continuously radiates energy. Therefore, an electron during its orbital motion, should go on radiating energy. Due to the loss of energy, the radius of its orbit should go on decreasing. Therefore, the electron should move along a spiral path and finally fall into the nucleus in a very short time, of the order of 10^{-16} s in the case of a hydrogen atom. Thus, the atom should be unstable. We exist because atoms are stable. (1 mark)

(2) If the electron moves along such a spiral path, the radius of its orbit would continuously decrease. As a result, the speed and frequency of revolution of the electron would go on increasing. The electron, therefore, would emit radiation of continuously changing frequency, and hence give rise to a continuous spectrum. However, atomic spectrum is a line spectrum. (1 mark)

Note : Show log calculations neatly.

Q. 8. Data : $n_g = 1.5$, $n_w = 1.333$

$$n_g = \frac{c}{v_g},\ n_w = \frac{c}{v_w} \quad \text{... (1)}$$

Since, $n_g > n_w$, $v_g < v_w$

$\therefore v_w - v_g = 2.505 \times 10^7$ m/s ... (Given)

From Eq. (1), $v_w - v_g = \frac{c}{n_w} - \frac{c}{n_g}$

$\therefore c\left(\frac{1}{n_w} - \frac{1}{n_g}\right) = 2.505 \times 10^7$ ($\frac{1}{2}$ mark)

$\therefore c\left(\frac{1}{1.333} - \frac{1}{1.5}\right) = 2.505 \times 10^7$ ($\frac{1}{2}$ mark)

$\therefore c\left(\frac{1.5 - 1.333}{1.333 \times 1.5}\right) = 2.505 \times 10^7$

$\therefore c = \frac{2.505 \times 10^7 \times 1.333 \times 1.5}{0.167}$

$= 30.00 \times 10^7$ m/s $= 3 \times 10^8$ m/s (1 mark)

This is the speed of light in air.

log 2.505	0.3988
log 1.333	+ 0.1249
log 1.5	+ 0.1761
	0.6998
log 0.167	− $\bar{1}$.2227
	1.4771
AL (1.4771) =	30.00

Q. 9. Let the alternating current of peak value i_0 and frequency $f = 1/T$ be

$i = i_0 \sin \omega t$ ... (1)

where $\omega = 2\pi f = \frac{2\pi}{T}$. The heat produced in the resistor of resistance R in one complete cycle is

$$H = \int_0^T i^2 R dt = R \int_0^T i_0^2 \sin^2 \omega t \, dt = i_0^2 R \int_0^T \sin^2 \omega t \, dt \quad \text{... (2)}$$

(1 mark)

Using the trigonometrical identity,

$\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$,

$$\int_0^T \sin^2 \omega t \, dt = \int_0^T \frac{1}{2} dt - \int_0^T \frac{\cos 2\omega t}{2} dt$$

$$= \frac{T}{2} - \frac{1}{2}\left[\frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{T}{2} - \frac{1}{4\omega}\left[\sin 2\left(\frac{2\pi}{T}\right)T - \sin 0\right]$$

$$= \frac{T}{2} - \frac{1}{4\omega}(0 - 0) = \frac{T}{2} \quad \text{... (3)}$$

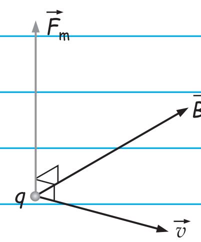
($\frac{1}{2}$ mark)

$$\therefore H = \frac{i_0^2 RT}{2} = i_{rms}^2 RT = \frac{i_{rms}^2 R}{f} \quad \text{... (4)}$$

($\frac{1}{2}$ mark)

Equation (4) is the required expression.

Q. 10. The magnetic force on a particle carrying a charge q and moving with a velocity $\vec{v}$ in a magnetic field of induction $\vec{B}$ is $\vec{F}_m = q\vec{v} \times \vec{B}$. At every instant, $\vec{F}_m$ is perpendicular to the linear velocity $\vec{v}$ and $\vec{B}$. Therefore, a non-zero magnetic force may change the direction of the velocity and the dot product $\vec{F}_m \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$.



$\vec{F}_m$ perpendicular to the plane containing $\vec{v}$ and $\vec{B}$

(1 mark)

But $\vec{F}_m \cdot \vec{v}$ is the power, i.e., the time rate of doing work. Hence, the work done by the magnetic force in every short displacement of the particle is zero. The work done by a force produces a change in kinetic energy. Zero work means no change in kinetic energy. Thus, although the magnetic force changes the direction of the velocity $\vec{v}$, it cannot change the linear speed and the kinetic energy of the particle. (1 mark)

Q. 11. Data : $N = 2000$, $A_i = 1.5 \times 10^{-4}$ m^2, $A_f = 0$, $B = 0.6$ T, $\Delta t = 0.3$ s

Initial flux, $N\Phi_i = NBA_i = 2000(0.6)(1.5 \times 10^{-4})$

$= 0.18$ Wb

Final flux, $N\Phi_f = 0$, since the coil is moved out of the field.

($\frac{1}{2}$ mark)

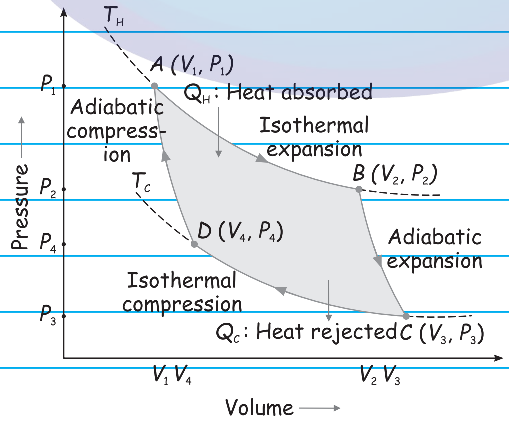
Induced emf, $e = -N\frac{\Delta \Phi_m}{\Delta t} = -N\frac{\Phi_f - \Phi_i}{\Delta t}$

($\frac{1}{2}$ mark)

$\therefore e = -\frac{0 - 0.18}{0.3}$ ($\frac{1}{2}$ mark)

$= 0.6$ V ($\frac{1}{2}$ mark)

Q. 12.



Carnot cycle (Diagram : 1 mark, Labelling : 1 mark)

Q. 13 The total energy of a particle of mass m executing SHM with frequency $f = \frac{\omega}{2\pi}$ and amplitude A is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi mf^2A^2 = \frac{2\pi mA^2}{T^2}$$

where ω is a constant in a particular case, $T = 2\pi/\omega$ is the period of the motion and $k = m\omega^2$ is the force constant.

Conclusions : The total energy of the particle is

(i) independent of its position x on the path and thus remains constant when m, ω and A are constant,

(ii) directly proportional to the force constant ($E \propto k$), ($\frac{1}{2}$ mark)

(iii) directly proportional to the mass of the particle ($E \propto m$),

($\frac{1}{2}$ mark)

(iv) directly proportional to the square of the amplitude ($E \propto A^2$),

($\frac{1}{2}$ mark)

(v) proportional to the square of the frequency f ($E \propto f^2$),

(vi) inversely proportional to the square of the period T ($E \propto \frac{1}{T^2}$).

($\frac{1}{2}$ mark)

Q. 14. Data : $C_1 = 8\ \mu F$, $C_2 = 8\ \mu F$, $C_3 = 4\ \mu F$, $V = 120$ V

Let C_S = equivalent capacity of the series combination of the capacitors

$$\therefore \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4}$$ ($\frac{1}{2}$ mark)

$$= \frac{1+1+2}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\therefore C_S = 2\ \mu F = 2 \times 10^{-6}\ F$$ ($\frac{1}{2}$ mark)

In series combination, the charge on each capacitor is the same. It is given by $Q = C_S V$. ($\frac{1}{2}$ mark)

Therefore, the charge on the $4 \mu F$ capacitor is

$$\therefore Q = 2 \times 10^{-6} \times 120$$

$$= 2.4 \times 10^{-4}\ \text{coulomb} = 240\ \mu C$$ ($\frac{1}{2}$ mark)

SECTION - C

Q. 15. Data : $A_t = 6\ min^{-1}g^{-1}$, $A_0 = 13.6\ min^{-1}g^{-1}$, $\lambda = 3.84 \times 10^{-12}\ s^{-1}$,

$1\ y = 3.154 \times 10^7\ s$

$A_t = A_0 e^{-\lambda t}$

$\therefore\ \lambda t = \ell n \frac{A_0}{A_t} = 2.303 \log \frac{A_0}{A_t}$ (1 mark)

$\therefore\ t = \frac{2.303}{\lambda}(\log A_0 - \log A_t)$

$= \frac{2.303}{3.84 \times 10^{-12}}(\log 13.6 - \log 6)$ (1 mark)

$= \frac{2.303 \times 10^{12}}{3.84}(1.1335 - 0.7782)$

$= \frac{2.303 \times 0.3553}{3.84} \times 10^{12}\ s$

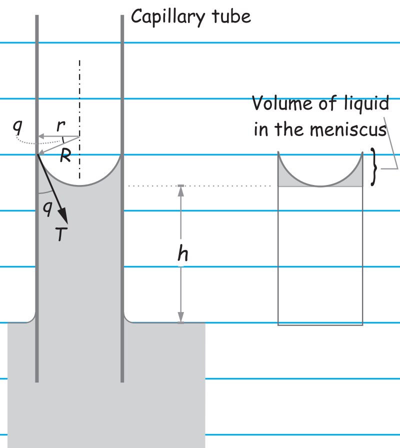
$= \frac{2.303 \times 35.53}{3.84 \times (3.154 \times 10^7\ s/y)} \times 10^{10}$

$= AL(0.8297) \times 10^3 = 6.756 \times 10^3\ y$

log 3.8400	0.5843
log 3.154	+ 0.4989
	1.0832
log 2.303	0.3623
log 35.53	+ 1.5506
	1.9129
	− 1.0832
	0.8297
AL (0.8297) =	6.756

The age of the given wood sample is 6756 years. (1 mark)

Q. 16 Consider a capillary tube of radius r partially immersed into a wetting liquid of density ρ. Let the capillary rise be h and θ the angle of contact at the edge of contact of the concave meniscus and glass as shown in the figure. If R is the radius of curvature of the meniscus, then from the figure, $r = R \cos \theta$. ($\frac{1}{2}$ mark)



Capillary rise (1 mark)

Surface tension T is the tangential force per unit length acting along the contact line. It is directed into the liquid making an angle θ with the capillary wall. We ignore the small volume of the liquid in the meniscus. The gauge pressure within the liquid at a depth h, i.e., at the level of the free liquid surface open to the atmosphere, is

$$p - p_0 = \rho g h \qquad \text{... (1)}$$ (½ mark)

By Laplace's law for a spherical membrane, this gauge pressure is

$$p - p_0 = \frac{2T}{R} \qquad \text{... (2)}$$ (½ mark)

$$\therefore\ h\rho g = \frac{2T}{R} = \frac{2T\cos\theta}{r}$$

$$\therefore\ \text{Capillary rise, } h = \frac{2T\cos\theta}{r\rho g} \qquad \text{... (3)}$$ (½ mark)

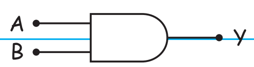
Equation (3) is the required expression.

Q. 17. The AND gate : It is a circuit with two or more inputs and one output in which the output signal is HIGH if and only if all the inputs are HIGH simultaneously. (1 mark)

The AND operation represents a logical multiplication.

Figure shows the 2-input AND gate logic symbol and the Boolean expression and the truth table for the AND function.

Logic symbol :



(½ mark)

Boolean expression :

$Y = A \cdot B$ (½ mark)

Truth Table : (1 mark)

Inputs		Output
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

Note : Convert the temperatures in degree Celsius to kelvin.

Q. 18. Data : $r = 2.5$ cm $= 2.5 \times 10^{-2}$ m, $a = 0.9$, $T_0 = 273 + 27 = 300$ K,

$T = 273 + 127 = 400$ K, $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$K^4

Emissivity, $e = a = 0.9$

The rate at which energy must be supplied is

$= e\sigma A(T^4 - T_0^4) = e\sigma(4\pi r^2)(T^4 - T_0^4)$ (1 mark)

$= (0.9)(5.67 \times 10^{-8})(4 \times 3.142)(2.5 \times 10^{-2})^2(400^4 - 300^4)$ (1 mark)

$= 3.142 \times 5.67 \times 22.5 \times 10^{-4}(4^2 + 3^2)(4^2 - 3^2)$

$= 3.142 \times 5.67 \times 22.5 \times 10^{-4}(25 \times 7)$

$= 3.142 \times 5.67 \times 22.5 \times 175 \times 10^{-4}$

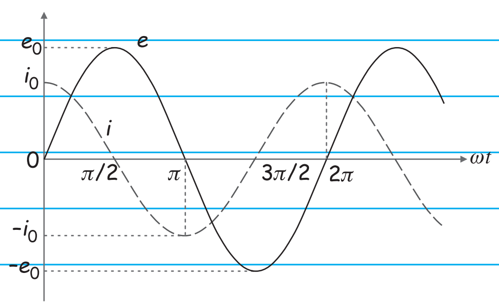
$= 7.015$ W (1 mark)

log 3.142	0.4972
log 5.67	+ 0.7536
log 22.5	+ 1.3522
log 175	+ 2.2430
	4.8460
AL (4.8460)	= 70150

Q. 19. (a) Since the current leads the emf (or, equivalently, the emf lags the current) by $\frac{\pi}{2}$ rad in phase, the device Y is a capacitor. ($\frac{1}{2}$ mark)

Its reactance is $X_C = \frac{1}{\omega C}$, where ω is the angular frequency of the applied emf and C is the capacitance of the capacitor. ($\frac{1}{2}$ mark)

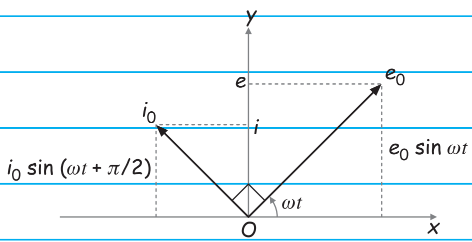
(b)



Graphs of e and i versus ωt for a purely capacitive AC circuit

(1 mark)

(c)



Phasor diagram for a purely capacitive circuit (1 mark)

Q. 20. Data : $T_f = 2\ T_i$, monatomic gas $\therefore\ \gamma = 5/3$

$P_i V_i^{\gamma} = P_f V_f^{\gamma}$ ... (adiabatic process) ($\frac{1}{2}$ mark)

For an ideal gas, $PV = nRT$ ($\frac{1}{2}$ mark)

$\therefore\ V = \frac{nRT}{P}$

$\therefore\ V_i = \frac{nRT_i}{P_i}$ and $V_f = \frac{nRT_f}{P_f}$ ($\frac{1}{2}$ mark)

$\therefore\ P_i \left(\frac{nRT_i}{P_i}\right)^{\gamma} = P_f \left(\frac{nRT_f}{P_f}\right)^{\gamma}$

$\therefore\ P_i^{1-\gamma}\ T_i^{\gamma} = P_f^{1-\gamma}\ T_f^{\gamma} \quad \therefore\ \left(\frac{T_f}{T_i}\right)^{\gamma} = \left(\frac{P_i}{P_f}\right)^{1-\gamma}$ ($\frac{1}{2}$ mark)

$\therefore\ \left(\frac{T_f}{T_i}\right)^{\gamma} = \left(\frac{P_f}{P_i}\right)^{\gamma - 1}$

$\therefore\ 2^{5/3} = \left(\frac{P_f}{P_i}\right)^{5/3 - 1} = \left(\frac{P_f}{P_i}\right)^{2/3}$ ($\frac{1}{2}$ mark)

$\therefore\ \frac{P_f}{P_i} = 2^{\frac{5}{3} \times \frac{3}{2}} = 2^{\frac{5}{2}} = \sqrt{2^5} = \sqrt{32}$

$= 4\sqrt{2} = 4 \times 1.414$

$\therefore\ \frac{P_f}{P_i} = 5.656$ ($\frac{1}{2}$ mark)

This is the ratio of the final pressure (P_f) to the initial pressure (P_i).

Q. 21. Consider two sound waves of equal amplitude (A) and slightly different frequencies n_1 and n_2 (with $n_1 > n_2$) propagating through the medium in the same direction and along the same line. These waves can be represented by the equations $y_1 = A \sin 2\pi n_1 t$ and $y_2 = A \sin 2\pi n_2 t$ at $x = 0$, where y denotes the displacement of the particle of the medium from its mean position. ($\frac{1}{2}$ mark)

By the principle of superposition of waves, the resultant displacement of the particle of the medium at the point at which the two waves arrive simultaneously is the algebraic sum

$y = y_1 + y_2 = A \sin 2\pi n_1 t + A \sin 2\pi n_2 t$

Now, $\sin C + \sin D$

$= 2 \sin \left(\frac{C+D}{2}\right) \cos \left(\frac{C-D}{2}\right)$

$\therefore y = 2A \sin \left[2\pi \left(\frac{n_1+n_2}{2}\right)t\right] \cos \left[2\pi \left(\frac{n_1-n_2}{2}\right)t\right]$ (½ mark)

$= 2A \cos \left[2\pi \left(\frac{n_1-n_2}{2}\right)t\right] \sin \left[2\pi \left(\frac{n_1+n_2}{2}\right)t\right]$

Let $R = 2A \cos \left[2\pi \left(\frac{n_1-n_2}{2}\right)t\right]$ and $n = \frac{n_1+n_2}{2}$

$\therefore y = R \sin 2\pi nt$ (½ mark)

The above equation shows that the resultant motion has amplitude $|R|$ which changes periodically with time. The period of beats is the period of waxing (maximum intensity of sound) or the period of waning (minimum intensity of sound). (½ mark)

The intensity of sound is directly proportional to the square of the amplitude of the wave. It is maximum (waxing) when $|R|$ becomes maximum;

$R = \pm 2A.$

$\therefore \cos \left[2\pi \left(\frac{n_1-n_2}{2}\right)t\right] = \pm 1$

$\therefore 2\pi \left(\frac{n_1-n_2}{2}\right)t = 0, \pi, 2\pi, 3\pi,$

$\therefore t = 0, \frac{1}{n_1-n_2}, \frac{2}{n_1-n_2}, \frac{3}{n_1-n_2}, \ldots$ ... (1) (½ mark)

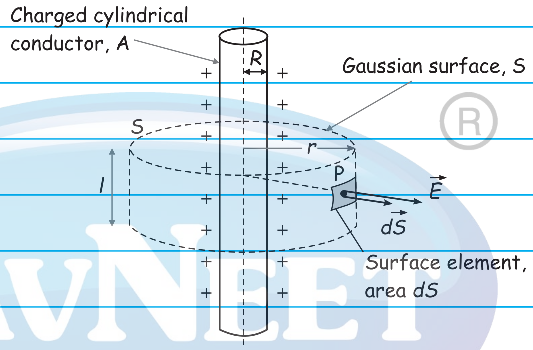
$\therefore$ Period of beats = period of waxing $= \frac{1}{n_1-n_2}$

$\therefore$ Beat frequency $= \frac{1}{\text{period of beats}} = n_1 - n_2$ (½ mark)

Q. 22. Consider an isolated cylindrical conductor A, of radius R and carrying a charge per unit length λ. We assume the conductor to be infinitely long. Consider a point P outside the conductor at a distance r from its axis as shown in the figure. To find the electric field intensity at P, we choose a cylindrical Gaussian surface S of radius r through P and coaxial with the conductor A. As λ is the charge per unit length of conductor A, the net charge enclosed by the Gaussian cylinder of length ℓ is

$Q = \lambda\ell$ ... (1) ($\frac{1}{2}$ mark)

A small element on the curved part of the Gaussian surface and containing P has an area dS.



(1 mark)

Electric field intensity at a point outside a uniformly charged cylindrical conductor assumed to be infinitely long

Charge is uniformly distributed over the outer surface of the cylindrical conductor. Then, by symmetry, the electric field intensity at any point outside the conductor is perpendicular to the cylinder axis. Hence, the component of the electric field intensity perpendicular to the plane circular faces of the Gaussian surface is zero. Therefore, the electric flux through these flat faces is zero.

By symmetry, the electric field intensity $\vec{E}$ at every point on the curved face of surface S is normal to the surface and has the same magnitude E. If the charge on conductor A is positive, $\vec{E}$ is directed along the outward drawn normal $d\vec{S}$.

The angle θ between $\vec{E}$ and $d\vec{S}$ being zero for every surface element, the electric flux through every element is

$d\Phi = \vec{E} \cdot d\vec{S} = EdS$

Therefore, the flux through the curved face of the Gaussian surface S is

$\Phi = \oint E\, dS = E \oint dS$ ... (2) ($\frac{1}{2}$ mark)

$\oint dS$ = area of the curved surface $= 2\pi r\ell$, where ℓ is the length of the cylinder as shown in the figure.

$\therefore\ \Phi = E \times 2\pi r\ell$ ... (3)

Then, by Gauss's theorem,

$\Phi = \frac{Q}{\varepsilon} = E \times 2\pi r\ell$ ... (4) ($\frac{1}{2}$ mark)

$\therefore\ E = \frac{\lambda\ell}{\varepsilon(2\pi r\ell)} = \frac{\lambda}{2\pi\varepsilon r} = \frac{\lambda}{2\pi k\varepsilon_0 r}$ ... (5) ($\frac{1}{2}$ mark)

where ε_0 is the permittivity of free space and $k = \frac{\varepsilon}{\varepsilon_0}$ is the relative permittivity (dielectric constant) of the surrounding medium.

This gives the magnitude of the electric field intensity in terms of the linear charge density λ. For positive λ, $\vec{E}$ is outward, while for negative λ, $\vec{E}$ is inward.

Note : Convert energy in electronvolt into joule.

Q. 23. Data : KE = 150 eV, $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J·s,

$m_e = 9.1 \times 10^{-31}$ kg

$KE = \frac{p^2}{2m_e}$

The momentum of the electron,

$p = \sqrt{2m_e KE}$ ($\frac{1}{2}$ mark)

$= \sqrt{2(9.1 \times 10^{-31})(150 \times 1.6 \times 10^{-19}\ J)}$ ($\frac{1}{2}$ mark)

$= \sqrt{9.1 \times 4.8 \times 10^{-48}}$

$= 6.609 \times 10^{-24}$ kg·m/s (1 mark)

log 9.1	0.9590
log 4.8	+0.6812
	1.6402
$\frac{1.6402}{2}$ =	0.8201
AL (0.8201) =	6.609

The associated wavelength,

$\lambda = \frac{h}{p}$ (½ mark)

$= \frac{6.63 \times 10^{-34}}{6.609 \times 10^{-24}}$

log 6.63	0.8215
log 6.609	$-$ 0.8201
	0.0014
AL (0.0014) =	1.003

$= 1.003 \times 10^{-10}$ m $= 0.1003$ nm (½ mark)

Q. 24. Data : $\mu = 0.025$ J/T, $B = 50$ mT $= 5 \times 10^{-2}$ T,

$\Delta K = 625\ \mu J = 6.25 \times 10^{-4}$ J

Change in potential energy,

$\Delta U = U_0 - U_\theta = -\mu B \cos 0° - (-\mu B \cos \theta)$

$= -\mu B(1 - \cos \theta)$ (½ mark)

By the principle of conservation of energy,

$\Delta K + \Delta U = 0$ (½ mark)

$\therefore \Delta K = -\Delta U = \mu B(1 - \cos \theta)$ (½ mark)

$6.25 \times 10^{-4} = (2.5 \times 10^{-2})(5 \times 10^{-2})(1 - \cos \theta)$ (½ mark)

$\therefore 6.25 = 12.5(1 - \cos \theta)$

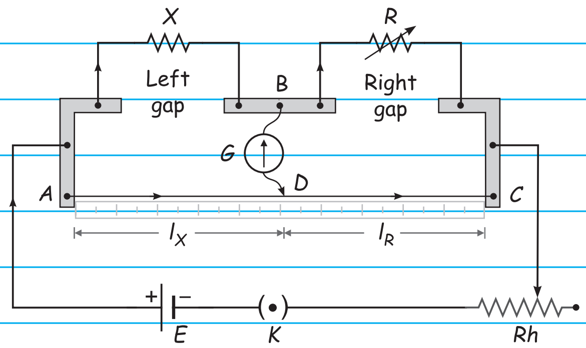
$\therefore (1 - \cos \theta) = \frac{6.25}{12.5} = 0.5 \qquad \therefore \cos \theta = 0.5$ (½ mark)

The initial angle between the dipole moment and the magnetic field,

$\theta = 60°$ (½ mark)

Note : Explain the construction in brief with a neat labelled circuit diagram. Then explain the working.

Q. 25. A metre bridge consists of a rectangular wooden board with two L-shaped thick metallic strips fixed along its three edges. A single thick metallic strip separates two L-shaped strips. A wire of length one metre and uniform cross-section is stretched on a metre scale fixed on the wooden board. The ends of the wire are fixed to the L-shaped metallic strips.



X : Unknown resistance, R : Resistance box,
AC : Uniform resistance wire, D : Null point,
E : Cell, K : Plug key, G : Galvanometer, Rh : Rheostat

A metre bridge (1 mark)

An unknown resistance X is connected in the left gap and a resistance box R is connected in the right gap as shown in the figure. One end of a centre-zero galvanometer (G) is connected to terminal B and the other end is connected to a pencil jockey.

A cell (E) of emf E, plug key (K) and rheostat (Rh) are connected in series between points A and C. ($\frac{1}{2}$ mark)

Working : Keeping a suitable resistance (R) in the resistance box, key K is closed to pass a current through the circuit. The jockey is tapped along the wire to locate the equipotential point D when the galvanometer shows zero deflection. The bridge is then balanced and point D is called the null point and the method is called as null deflection method. The distances ℓ_X and ℓ_R of the null point from the two ends of the wire are measured. ($\frac{1}{2}$ mark)

According to the principle of Wheatstone's network,

$$\therefore \frac{X}{R} = \frac{R_{AD}}{R_{DC}} \quad \text{... (1)}$$ ($\frac{1}{2}$ mark)

$$\therefore R_{AD} = \rho\frac{\ell_X}{A} \text{ and } R_{DC} = \rho\frac{\ell_R}{A} \quad \text{... (2)}$$

where ℓ_X and ℓ_R are the lengths of the wires AD and DC, ρ is the resistivity of the material of the wire and A is the area of cross-section of the wire.

$$\therefore \frac{X}{R} = \frac{R_{AD}}{R_{DC}} = \frac{\rho\ell_X/A}{\rho\ell_R/A}$$

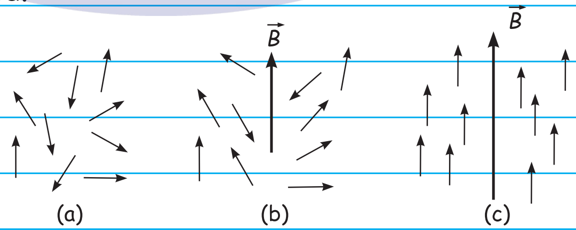
$\therefore \frac{X}{R} = \frac{\ell_X}{\ell_R}$

$\therefore X = \frac{\ell_X}{\ell_R} \times R$ ($\frac{1}{2}$ mark)

As R, ℓ_X and ℓ_R are known, the unknown resistance X can be calculated.

Q. 26. Paramagnetism depends on the presence of permanent atomic or molecular magnetic dipole moments. The inherent net atomic magnetic moment results from a particular combination of the spin and orbital magnetic moments of its electrons.

The spin magnetic moments of the electrons in matter are affected by the internal magnetic field created by the magnetic moments of surrounding electrons. This internal field, $\sim 10^{-2}$ T to 10^{-1} T, causes the spin magnetic moments to precess about the field direction. At normal temperature; the thermal motion of the electrons produces constant fluctuations in the internal field so that the spin magnetic moments have random directions, Fig. (a). In the absence of an external magnetizing field, therefore, a paramagnetic material is not magnetized. (1 mark)



Magnetic dipole moments in a paramagnetic sample

(a) randomly directed in the absence of a magnetizing field

(b) partial alignment on the application of an external field

(c) aligned to saturation at very low temperature or strong field

(1 mark)

When the applied field strength is greater than that of the internal field, the spin magnetic moments tend to align parallel to the external field direction. But the randomizing effect of thermal agitation prevents complete alignment, Fig. (b). Therefore, at room temperature, when a paramagnetic material is placed in a magnetic field, it is weakly magnetized in the direction of the magnetizing field.

($\frac{1}{2}$ mark)

If the external field is very large or the temperature is very low, the magnetic dipole moments are effectively aligned parallel to the field so as to have the least magnetic potential energy and the magnetization reaches saturation, Fig. (c). ($\frac{1}{2}$ mark)

SECTION - D

Q. 27. (a) Consider n moles of an ideal gas in a container of volume V. If m is the mass of a gas molecule and v_{rms} is the root-mean-square speed of the gas molecules, then, by the kinetic theory, the pressure exerted by the gas is

$$P = \frac{1}{3}\frac{Nm}{V}v^2_{rms} \quad \text{... (1)}$$ ($\frac{1}{2}$ mark)

where N is the number of molecules of the gas; $N = nN_A$, where N_A is the Avogadro number.

$$\therefore PV = \frac{1}{3}Nmv^2_{rms} = \frac{2}{3}N\left(\frac{1}{2}mv^2_{rms}\right) \quad \text{... (2)}$$

The equation of state of an ideal gas is

$$PV = nRT \quad \text{... (3)}$$ ($\frac{1}{2}$ mark)

$$\therefore \frac{2}{3}N\left(\frac{1}{2}mv^2_{rms}\right) = nRT$$

$$\therefore \frac{1}{2}mv^2_{rms} = \frac{3}{2}\frac{n}{N}RT = \frac{3}{2}\left(\frac{N/N_A}{N}\right)RT = \frac{3}{2}\frac{R}{N_A}T \quad \text{... (4)}$$ ($\frac{1}{2}$ mark)

The left-hand side is the average kinetic energy per molecule and $\frac{R}{N_A} = k_B$, the Boltzmann constant.

$$\therefore \text{Average KE per molecule} = \frac{3}{2}k_BT \quad \text{... (5)}$$ ($\frac{1}{2}$ mark)

Note : The areas appear in a ratio, so there is no need to change the units as long as they are same.

(b) Data : $A_1 = 25\ cm^2$, $A_2 = 1000\ cm^2$, $F_1 = 50\ N$

By Pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore F_2 = F_1\frac{A_2}{A_1}$$ ($\frac{1}{2}$ mark)

$$= (50\ N) \times \frac{1000\ cm^2}{25\ cm^2}$$ ($\frac{1}{2}$ mark)

$$= 2000\ N$$ (1 mark)

This is the weight on the output piston.

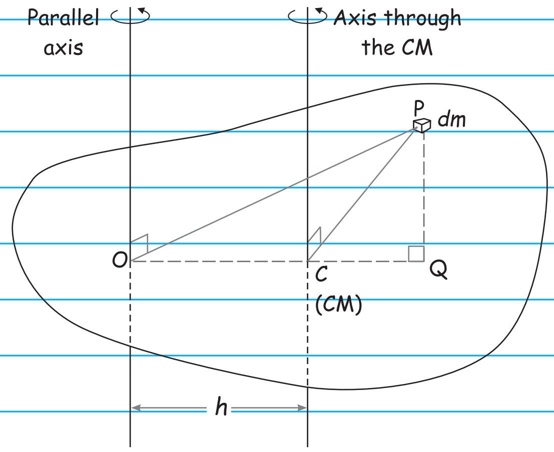
Q. 28. Theorem of parallel axis : The moment of inertia of a body about an axis is equal to the sum of (i) its moment of inertia about a parallel axis through its centre of mass and (ii) the product of the mass of the body and the square of the distance between the two axes.

(1 mark)

Proof : Let I_{CM} be the moment of inertia (MI) of a body of mass M about an axis through its centre of mass C, and I its MI about a parallel axis through any point O. Let h be the distance between the two axes.

Consider an infinitesimal mass element dm of the body at a point P. It is at a perpendicular distance CP from the rotation axis through C and a perpendicular distance OP from the parallel axis through O. The MI of the element about the axis through C is $CP^2\,dm$. Therefore, the MI of the body about the axis through the CM is $I_{CM} = \int CP^2\,dm$. Similarly, the MI of the body about the parallel axis through O is $I = \int OP^2\,dm$.

($\frac{1}{2}$ mark)



Theorem of parallel axis ($\frac{1}{2}$ mark)

Draw PQ perpendicular to OC produced, as shown in the figure. Then, from the figure,

$I = \int OP^2\, dm$

$= \int (OQ^2 + PQ^2)\, dm$

$= \int [(OC + CQ)^2 + PQ^2]\, dm$

$= \int (OC^2 + 2OC \cdot CQ + CQ^2 + PQ^2)\, dm$

$= \int (OC^2 + 2OC \cdot CQ + CP^2)\, dm$ $(\because CQ^2 + PQ^2 = CP^2)$

$= \int OC^2\, dm + \int 2OC \cdot CQdm + \int CP^2\, dm$

$= OC^2 \int dm + 2OC \int CQdm + \int CP^2\, dm$ (1 mark)

Since, $OC = h$ is constant and $\int dm = M$ is the mass of the body,

$I = Mh^2 + 2h \int CQdm + I_{CM}$

Now, from the definition of centre of mass, the integral $\int CQdm$ gives mass M times a coordinate of the CM with respect to the origin C. Since C is itself the CM, this coordinate is zero and so also the integral.

$\therefore I = I_{CM} + Mh^2$

This proves the theorem of parallel axis. (1 mark)

Q. 29. (a) The Rayleigh criterion for the limit of resolution for two linear objects : Consider, two self luminous objects or slits separated by some distance. Let λ be the wavelength of the light and a the width of the slits. As per the Rayleigh criterion, the first minimum of the diffraction pattern of one of the sources should coincide with the central maximum of the other. Thus, it is at the just resolved condition. (1 mark)

The angular separation $d\theta$ (position) of the first principal minimum is,

$$d\theta = \frac{\lambda}{a} \quad \text{... (1)}$$ ($\frac{1}{2}$ mark)

This angular separation between the two objects must be minimum as this minimum coincides with the central maximum of the other. This is called the limit of resolution of that instrument. It is written as,

limit of resolution, $d\theta = \frac{\lambda}{a}$

Minimum separation between the two linear objects that are just resolved, at distance D from the instrument is,

$$y = D\,(d\theta) = \frac{D\lambda}{a} \quad \text{... (2)}$$

It is the distance of the first minimum from the centre. ($\frac{1}{2}$ mark)

(b) Data : $n_g = 1.732$

$n_g = \tan \theta_B$

$\therefore \theta_B = \tan^{-1}(1.732) = 60°$ ($\frac{1}{2}$ mark)

This is the polarizing angle.

$n_g = \frac{\sin \theta_B}{\sin r_P}$ ($\frac{1}{2}$ mark)

$\therefore \sin \theta_r = \frac{\sin \theta_B}{n_g}$

$\therefore \sin \theta_r = \frac{\sin 60°}{1.732} = \frac{0.8660}{1.732}$ ($\frac{1}{2}$ mark)

$\therefore \theta_r = \sin^{-1}\left(\frac{0.8660}{1.732}\right)$

$= \sin^{-1}(0.5) = 30°$ ($\frac{1}{2}$ mark)

This is the angle of refraction.

Note : Unless specifically mentioned, you may draw a step-up or step-down transformer.

Q. 30. Principle : A transformer works on the principle that a changing current through one coil creates a changing magnetic flux through an adjacent coil which in turn induces an emf and a current in the second coil. ($\frac{1}{2}$ mark)

Construction : A transformer consists of two coils, primary and secondary, wound on two arms of a rectangular frame called the core.

(1) Primary coil : It consists of an insulated copper wire wound on one arm of the core. Input voltage is applied at the ends of this coil.

In a step-up transformer, thick copper wire is used for primary coil. In a step-down transformer, thin copper wire is used for primary coil. ($\frac{1}{2}$ mark)

(2) Secondary coil : It consists of an insulated copper wire wound on the other arm of the core. The output voltage is obtained at the ends of this coil.

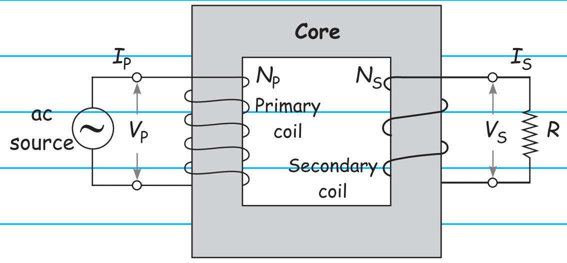
In a step-up transformer, thin copper wire is used for secondary coil. In a step-down transformer, thick copper wire is used for secondary coil. ($\frac{1}{2}$ mark)

(3) Core : It consists of thin rectangular frames of soft iron stacked together, but insulated from each other. A core prepared by stacking thin sheets rather than using a single thick sheet reduces eddy currents. ($\frac{1}{2}$ mark)

An alternating emf V_P from an ac source is applied across the primary coil of a transformer. This sets up an alternating current I_P in the primary circuit and also produces an alternating magnetic flux through the primary coil such that

$$V_P = -N_P \frac{d\Phi_P}{dt},$$

where N_P is the number of turns of the primary coil and Φ_P is the magnetic flux through each turn. ($\frac{1}{2}$ mark)



($\frac{1}{2}$ mark)

Assuming an ideal transformer (i.e., there is no leakage of magnetic flux), the same magnetic flux links both the primary and the secondary coils, i.e., $\Phi_P = \Phi_S$.

As a result, the alternating emf induced in the secondary coil,

$$V_S = -N_S \frac{d\Phi_S}{dt} = -N_S \frac{d\Phi_P}{dt}$$

where N_S is the number of turns of the secondary coil. ($\frac{1}{2}$ mark)

If the secondary circuit is completed by a resistance R, the secondary current is $I_S = V_S/R$, assuming the resistance of the coil to be far less than R. Ignoring power losses, the power delivered to the primary coil equals that taken out of the secondary coil, so $V_P I_P = V_S I_S$.

$$\therefore \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

which is the required expression. ($\frac{1}{2}$ mark)

Q. 31. (a) If A is the amplitude of a particle executing linear SHM, in the usual notation,

$$v_1 = \omega\sqrt{A^2 - x_1^2} \quad \text{... (1)}$$

and $v_2 = \omega\sqrt{A^2 - x_2^2}$ ... (2) ($\frac{1}{2}$ mark)

$$\therefore v_2^2 - v_1^2 = \omega^2 (A^2 - x_2^2 - A^2 + x_1^2)$$

$$= \omega^2 (x_1^2 - x_2^2)$$

But $\omega = 2\pi/T$, where T is the period of SHM.

$$\therefore v_2^2 - v_1^2 = \frac{4\pi^2}{T^2}(x_1^2 - x_2^2)$$

$$\therefore T^2 = 4\pi^2 \left(\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}\right)$$

$$\therefore T = 2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}} \quad \text{... (3)}$$ ($\frac{1}{2}$ mark)

Also, from Eqs. (1) and (2),

$$\therefore v_1^2 = \omega^2 (A^2 - x_1^2) \text{ and } v_2^2 = \omega^2 (A^2 - x_2^2)$$

$$\therefore \frac{v_2^2}{v_1^2} = \frac{A^2 - x_2^2}{A^2 - x_1^2}$$ ($\frac{1}{2}$ mark)

$$\therefore v_2^2 A^2 - v_2^2 x_1^2 = v_1^2 A^2 - v_1^2 x_2^2$$

$$\therefore A^2(v_2^2 - v_1^2) = v_2^2 x_1^2 - v_1^2 x_2^2$$

$$\therefore A^2 = \frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}$$

$$\therefore A = \sqrt{\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}$$ ($\frac{1}{2}$ mark)

(b) Data : $R = 250\ \Omega$, $E = 2V$, $r = 12\ \Omega$

The voltmeter reading, $V = IR$ ($\frac{1}{2}$ mark)

$$= \left(\frac{E}{R + r}\right) R$$ ($\frac{1}{2}$ mark)

$$= \left(\frac{2}{250 + 12}\right) 250$$ ($\frac{1}{2}$ mark)

$$= \frac{500}{262} = 1.908\ V \approx 1.9\ V$$ ($\frac{1}{2}$ mark)

log 500	2.6990
log 262	− 2.4183
	0.2807
AL (0.2807)	= 1.908