2021

NAVNEET PRACTICE PAPERS

COMMERCE

STANDARD XII

Based on the Board's New Textbooks and Paper Pattern

Updated as per the portions omitted from the syllabus for the year 2020-21

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MATHEMATICS AND STATISTICS

EVALUATION PLANS

1. (a) Theory/Written Examination : **80 Marks**

(b) Practical Examination : **20 Marks**

Total : 100 Marks

2. Question Paper Pattern for the Theory/Written Examination :

(a) For Mathematics and Statistics (Commerce) there will be one question paper divided into two sections, viz. Section – I and Section – II. Students should write the answers of both sections in the same answer book.

Section I : **40 marks**

Section II : **40 marks**

Total : 80 marks

(b) Each section will have three main questions as follows :

SECTION – I

Question No.	Question Type	Marks
Q. 1. (A)	6 Multiple Choice Questions (MCQ) (1 mark each)	06
(B)	3 True/False type Questions (1 mark each)	03
(C)	3 Fill in the blanks type Questions (1 mark each)	03
Q. 2. (A)	Solve any 2 out of 3 (3 marks each)	06
(B)	Solve any 2 out of 3 (4 marks each)	08
Q. 3. (A)	Solve any 2 out of 3 (3 marks each)	06
(B)	Solve any 1 out of 2 (4 marks each)	04
(C)	Solve any 1 out of 2 (Activity) (4 marks each)	04

SECTION – II

Question No.	Question Type	Marks
Q. 4. (A)	6 Multiple Choice Questions (MCQ) (1 mark each)	06
(B)	3 True/False type Questions (1 mark each)	03
(C)	3 Fill in the blanks type Questions (1 mark each)	03
Q. 5. (A)	Solve any 2 out of 3 (3 marks each)	06
(B)	Solve any 2 out of 3 (4 marks each)	08

Question No.	Question Type	Marks
Q. 6. (A)	Solve any 2 out of 3 (3 marks each)	06
(B)	Solve any 1 out of 2 (4 marks each)	04
(C)	Solve any 1 out of 2 (Activity) (4 marks each)	04

3. Chapterwise distribution of marks in the Question Paper :

SECTION – I

Sr. No.	Chapters	Marks with Options
1.	Mathematical Logic	08
2.	Matrices	08
3.	Differentiation	07
4.	Applications of Derivatives	09
5.	Integration	07
6.	Definite Integration	05
7.	Application of Definite Integration	04
8.	Differential Equations and Applications	10
	Total Marks	**58**

SECTION – II

Sr. No.	Chapters	Marks with Options
1.	Commission, Brokerage and Discount	06
2.	Insurance and Annuity	04
3.	Linear Regression	08
4.	Time Series	07
5.	Index Numbers	07
6.	Linear Programming	06
7.	Assignment Problem and Sequencing	09
8.	Probability Distributions	11
	Total Marks	**58**

4. Scheme for the conduct of Practical Examination :

There will be Practical Examination based on topics in Part I and Part II of the textbooks for 20 marks.

Distribution of Marks :

1. Journal	5 marks
2. Problem solving (Three Problems out of four practical problems each of 5 marks)	15 marks
Total	**20 marks**

NON-EVALUATIVE PORTION FOR THE ACADEMIC YEAR 2020-21 AS DECLARED ON 22 – 07 – 2020

Part – 1

1. Mathematical Logic :

Solved Examples on page no.
4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 19, 20, 22, 23, 24, 25, 26, 27.

Activities on page no. 2, 14, 34

2. Matrices :

Solved Examples on page no.
39, 43, 44, 45, 48, 49, 50, 52, 53, 54, 55, 61, 62, 64, 65, 66, 67, 69, 70, 71, 74, 75, 76, 77, 78, 79.

Activities on page no. 86, 87, 88.

3. Differentiation :

Solved Examples on page no.
90, 91, 92, 93, 94, 95, 96, 97, 98.

Activities on page no. 101, 102.

4. Applications of Derivatives :

Solved Examples on page no. 104, 105, 106, 108, 109, 110, 111, 112.

Activities on page no. 114, 115.

5. Integration :

Solved Examples on page no.
117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135.

Activities on page no. 135, 139, 140.

6. Definite Integration :

Solved Examples on page no. 141, 142, 143, 144, 145, 146, 147.

Activities on page no. 150, 151.

7. Applications of Definite Integration :

Solved Examples on page no. 154, 155, 156, 157.

Activities on page no. 158, 159.

8. Differential Equations and Applications :

Solved Examples on page no. 162, 163, 164, 166, 167, 168, 169, 170.

Activities on page no. 173.

Part – II

1. Commission, Brokerage and Discount :

Solved Examples on page no. 1, 2, 3, 4, 7, 8, 9, 10.

Activities on page no. 14, 15

2. Insurance and Annuity :

Solved Examples on page no. 17, 18, 19, 23, 24, 25, 26, 27.

Activities on page no. 32, 33.

3. Linear Regression :

Solved Examples on page no. 39, 40, 41, 44, 45, 46, 48, 49.

Activities on page no. 54, 55, 56.

4. Time Series :

Solved Examples on page no. 59, 62, 64, 65, 66.

Activities on page no. 70, 71.

5. Index Numbers :

Solved Examples on page no. 75, 76, 77, 79, 80, 81, 84, 85, 86.

Activities on page no. 94.

6. Linear Programming :

Solved Examples on page no. 96, 97, 100, 101.

Activities on page no. 105, 106, 107.

7. Assignment Problem and Sequencing :

Solved Examples on page no. 113, 114, 115, 118, 121, 122, 123, 124.

Activities on page no. 130, 131.

8. Probability Distribution :

Solved Examples on page no.
136, 137, 138, 139, 140, 142, 143, 144, 147, 149, 150, 151, 152.

(For detailed information please refer to Board's Website.)

PART 1

MODEL PRACTICE PAPER
(WITH SOLUTION AND MARKING SCHEME)

MATHEMATICS AND STATISTICS

Time : 3 Hours] **[Max. Marks : 80**

General Instructions :

(1) All questions are compulsory.

(2) Figures to the right indicate full marks.

(3) There are 6 questions divided into two sections.

(4) Write answers of Section I and Section II in the same answer book.

(5) Use of logarithmic table is allowed. Use of calculator is not allowed.

(6) For LPP, graph paper is not necessary. Only rough sketch of graph is expected.

(7) Start answer to each question on a new page.

SECTION – I

Q. 1. (A) Select and write the most appropriate answer from the given alternatives for each sub-question : **[6]**

(i) The statement $(\sim p \wedge q) \vee \sim q$ is equivalent to

(a) $p \vee q$ (b) $p \wedge q$ (c) $\sim(p \vee q)$ (d) $\sim(p \wedge q)$ (1)

(ii) If A is a 2×2 matrix such that

$A(\text{adj. } A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then $|A| = \ldots\ldots\ldots$

(a) 0 (b) 5 (c) 10 (d) 25 (1)

(iii) If $y = \log\left(\frac{e^x}{x^2}\right)$, then $\frac{dy}{dx} = \ldots\ldots\ldots$

(a) $\frac{2-x}{x}$ (b) $\frac{x-2}{x}$ (c) $\frac{e-x}{ex}$ (d) $\frac{x-e}{ex}$ (1)

(iv) The equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$ is

(a) $2x - y = 0$ (b) $2x + y - 5 = 0$ (c) $2x - y - 1 = 0$ (d) $x + y - 1 = 0$ (1)

(v) $\int \frac{dx}{x-x^2} = \dots\dots\dots$

(a) $\log|x| - \log|1-x| + c$ (b) $\log|1-x^2| + c$

(c) $-\log|x| + \log|1-x| + c$ (d) $\log|x-x^2| + c$ (1)

(vi) $\int_2^3 \frac{x}{x^2-1}\, dx = \dots\dots\dots$

(a) $\log\left(\frac{8}{3}\right)$ (b) $-\log\left(\frac{8}{3}\right)$ (c) $\frac{1}{2}\log\left(\frac{8}{3}\right)$ (d) $-\frac{1}{2}\log\left(\frac{8}{3}\right)$ (1)

(B) State whether the following statements are *True* or *False* : **[3]**

(i) If $\int \frac{x-1}{(x+1)(x-2)}\, dx = A\log|x+1| + B\log|x-2| + c$, then $A+B=1$. (1)

(ii) A_1 is the area enclosed by $y=f(x)$, $x=a$, $x=b$ and X-axis, A_2 is the area enclosed by $y=f(x)$, $x=c$, $x=d$ and X-axis. If $A_1=A_2$, then $a=c$, $b=d$. (1)

(iii) Order and degree of a differential equation are always positive integers. (1)

(C) Fill in the following blanks : **[3]**

(i) Truth value of : If $x=2$, then $x^2=-4$, is .... (1)

(ii) If $f'(x) = \frac{1}{x} + x$ and $f(1) = \frac{5}{2}$, then $f(x) = \log x + \frac{x^2}{2} + \dots\dots\dots$ (1)

(iii) The order of highest derivative occurring in the differential equation is called .... of the differential equation. (1)

Q. 2. (A) Attempt *any Two* of the following : **[6]**

(i) Examine whether the following statement pattern is a tautology or a contradiction or a contingency :

$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$ (3)

(ii) Solve the following equations by the method of reduction :

$x+2y+z=8$, $2x+3y-z=11$, $3x-y-2z=5$. (3)

(iii) If $e^x + e^y = e^{x+y}$, then show that $\frac{dy}{dx} = -e^{y-x}$ (3)

(B) Attempt *any Two* of the following : **[8]**

(i) Find MPC, MPS, APC and APS, if the expenditure E_C of a person with income I is given as $E_C = (0.0003)\,I^2 + (0.075)I$, when $I=1000$. (4)

(ii) Evaluate : $\int \frac{x}{(x-1)^2(x+2)}\, dx$. (4)

(iii) Solve the following differential equation :

$(x^2-y^2)dx + 2xy\, dy = 0$. (4)

Q. 3. (A) Attempt *any Two* of the following : **[6]**

(i) Write the converse, inverse and contrapositive of the following statement :
"If he studies, then he will go to college." (3)

(ii) Find $\dfrac{d^2y}{dx^2}$, if $y = 2at$, $x = at^2$. (3)

(iii) Find the area between the parabolas $y^2 = 7x$ and $x^2 = 7y$. (3)

(B) Attempt *any One* of the following : **[4]**

(i) Find the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ by using elementary row transformations. (4)

(ii) Evaluate : $\displaystyle\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\, dx.$ (4)

(C) Attempt *any One* of the following : **[4]**

(i) The rectangle has area of 50 cm^2. Find its dimensions for least perimeter.

Solution : Let x cm and y cm be the length and breadth of the rectangle.

Then its area is $xy = 50$.

$\therefore\ y = \dfrac{50}{\square}$

Perimeter of the rectangle $= 2\,(x + y)$

$$= 2\left(x + \frac{50}{\square}\right)$$

Let $f(x) = 2\left(x + \dfrac{50}{\square}\right)$

Then $f'(x) = 2\left(1 - \dfrac{50}{\square}\right)$

and $f''(x) = 2\left(0 + \square\right) = \dfrac{200}{\square}$

Now, $f'(x) = 0$, if $1 - \dfrac{50}{\square} = 0$

i.e. if $x^2 = \square$

i.e. if $x = \pm \square$

But x is not negative.

$\therefore\ x = \square$ and $f''\square = \dfrac{200}{\square} > 0$

$\therefore$ by the second derivative test, f is minimum at $x = \square$

When $x = \square$, $y = \dfrac{50}{\square} = \square$

$\therefore\ x = \square$ cm, $y = \square$ cm

Hence, the rectangle is a square of side $\square$ cm. **(4)**

(ii) Solve : $\dfrac{dy}{dx} + 2xy = x$.

Solution : $\dfrac{dy}{dx} + 2xy = x$ ... (1)

This is the linear differential equation of the form $\dfrac{dy}{dx} + P \cdot y = Q$, where

$P = \square$, $Q = \square$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \square dx} = e^{\square}$

$\therefore$ the solution of (1) is given by

$y \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.}) dx + c$.

$\therefore\ y \cdot \square = \int \square \cdot e^{\square} dx + c$ ... (2)

Put $x^2 = t$ $\therefore\ 2x\,dx = dt$

$\therefore\ x\,dx = \dfrac{1}{2}\,dt$

$\therefore$ (2) becomes

$y \cdot \square = \dfrac{1}{2} \int \square\ dt + c$

$\therefore\ y \cdot \square = \dfrac{1}{2} \square + c$

$\therefore\ y \cdot \square = \dfrac{1}{2} e^{x^2} + c$

This is the general solution. **(4)**

SECTION – II

Q. 4. (A) Select and write the most appropriate answer from the given alternatives for each sub-question : [6]

(i) The market price is also called as

(a) cost price (b) selling price (c) list price (d) invoice price (1)

(ii) The cost of living index number using weighted relative method is given by

(a) $\frac{\Sigma IW}{\Sigma W}$ (b) $\Sigma\left(\frac{W}{IW}\right)$ (c) $\left(\frac{\Sigma W}{\Sigma IW}\right)$ (d) $\Sigma\left(\frac{IW}{W}\right)$ (1)

(iii) If $X \sim B\left(20, \frac{1}{10}\right)$, then $E(x)$ is

(a) 2 (b) 5 (c) 4 (d) 3 (1)

(iv) The corner points of the feasible region are (0, 0), (2, 0), $\left(\frac{12}{7}, \frac{1}{7}\right)$ and (0, 1), then the point of maximum $z = 6.5x + y = 15$ is

(a) (0, 0) (b) (2, 0) (c) $\left(\frac{12}{7}, \frac{3}{7}\right)$ (d) (0, 1) (1)

(v) The job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine, then optimal sequence is

(a) CDAB (b) DBCA (c) BCDA (d) ABCD (1)

(vi) Given p.d.f. of a continuous r.v. X as

$f(x) = \frac{x^2}{3}$, for $-1 < x < 2$

$= 0$, otherwise, then $F(1)$ is

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{3}{9}$ (d) $\frac{4}{9}$ (1)

(B) State whether the following statements are *True* or *False* : [3]

(i) b_{yx} and b_{xy} are independent of change of origin and scale. (1)

(ii) The region represented by the inequations $x \leqslant 0$ and $y \leqslant 0$ lies in the first quadrant. (1)

(iii) If r.v. X assumes the values 1, 2, 3, ..., 9 with equal probabilities, then $E(X) = 5$. (1)

(C) Fill in the following blanks : [3]

(i) The person who receives annuity is called .... (1)

(ii) The constraint that a factory has to employ more women (y) than men (x) is given by .... (1)

(iii) The value of discrete r.v. are generally obtained by .... (1)

Q. 5. (A) Attempt *any Two* of the following : [6]

(i) The following are the marks obtained by the students in Economics (X) and Mathematics (Y) :

X	59	60	61	62	63
Y	78	82	82	79	81

Find the regression equation of Y on X. (3)

(ii) Obtain trend values for the following data using 5-yearly moving averages :

Year	1974	1975	1976	1977	1978	1979	1980	1981	1982
Production	0	4	9	9	8	5	4	8	10

(3)

(iii) Find y, if the Price Index Number by Simple Aggregate Method is 120, taking 1995 as base year :

Commodity	A	B	C	D
Price (in ₹) in 1995	95	y	80	35
Price (in ₹) in 2003	116	74	92	42

(3)

(B) Attempt *any Two* of the following : [8]

(i) A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B, C in the order ABC. The time required in hours for each process is given in the following table :

Type	1	2	3	4	5
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Solve the problem for minimizing the total elapsed time. (4)

(ii) The following table gives the production of steel (in millions of tonnes) for years 1976 to 1986 :

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Production	0	4	4	2	6	8	5	9	4	10	10

Fit a trend line to the above data by the method of least squares. Also, obtain the trend value for the year 1990. (4)

(iii) A company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B :

Chemical → Raw Material ↓	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of A and one unit of B respectively. Formulate the problem as LPP to maximize profit. (4)

Q. 6. (A) Attempt *any Two* of the following : **[6]**

(i) Calculate Laspeyre's and Paasche's Price Index Number for the following data :

Commodity	Base Year		Current Year	
	Price p_0	Quantity q_0	Price p_1	Quantity q_1
I	8	30	12	25
II	10	42	20	16

(3)

(ii) A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below :

Salesmen	Districts			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

Find the assignment of salesman to various districts which will yield maximum profit. (3)

(iii) A player tosses two coins. He wins ₹ 10 if 2 heads appear, ₹ 5 if 1 head appears and ₹ 2 if no head appears. Find the expected value and variance of winning amount. (3)

(B) Attempt *any One* of the following : **[4]**

(i) For a bivariate data :

$\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.2$ and $b_{xy} = -0.3$. Find

(a) Correlation coefficient between X and Y.

(b) Estimate of Y for X = 50.

(c) Estimate of X for Y = 25. (4)

(ii) The number of complaints which a bank manager receives per day follows a Poisson distribution with parameter m = 4. Find the probability that the manager receives (a) only two complaints on a given day (b) at most two complaints on a given day. Use $e^{-4} = 0.0183$. (4)

(C) Attempt *any One* of the following : **[4]**

(i) **Complete the following activity :**

Face value (SD) = ₹ 7000, $r = 5\%$, Cash value = ₹ 6930,

BD = SD − ☐ = ☐ − ☐ = ₹ 70.

Date of drawing the bill = 14th April 2019

Date of discounting the bill = 6th July 2019.

$$BD = \frac{SD \times n \times r}{100}$$

$$\therefore\ 70 = 7000 \times \frac{n}{\square} \times \square$$

$$\therefore\ n = \frac{70}{350} \text{ years}$$

$\therefore\ n =$ ☐ = ☐ days.

For legal due date, 73 days are counted from due date of discounting the bill i.e. from 6th July 2019.

July 2019	Aug. 2019	Sept. 2019	Total
☐	31	☐	73

Legal due date : 17th Sept. 2019.

Nominal due date is ☐

Period of the bill is ☐ − ☐ = 5 months. (4)

(ii) **Complete the following activity :**

Present value P = ₹ 10000

Accumulated value A = ₹ 20,000 $\quad r = 12\%$

$i = \boxed{} = \boxed{} = 0.12$

C = Payment of each annuity

$$\boxed{\frac{}{}} - \boxed{\frac{}{}} = \frac{\boxed{}}{C}$$

$$\therefore\ 0.0001 - \boxed{} = \frac{0.12}{C}$$

$$\therefore\ 0.0001 - \boxed{} = \frac{0.12}{C}$$

$$\therefore\ C = \frac{0.12}{\boxed{}}$$

$\therefore$ C = ₹ 2400. (4)

SECTION - I

Q. 1. (A)

(i) (d) $\sim(p \wedge q)$ (1 mark)

(ii) (b) 5 (1 mark)

(iii) (b) $\frac{x-2}{x}$ (1 mark)

(iv) (a) $2x - y = 0$ (1 mark)

(v) (a) $\log|x| - \log|1-x| + c$ (1 mark)

(vi) (c) $\frac{1}{2}\log\left(\frac{8}{3}\right)$ (1 mark)

Q. 1. (B)

(i) True (1 mark)

(ii) False (1 mark)

(iii) True (1 mark)

Q. 1. (C)

(i) False (F) (1 mark)

(ii) 2 (1 mark)

(iii) order (1 mark)

Q. 2. (A)

(i)

1	2	3	4	5	6	7
p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge \sim q$	$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$
T	T	F	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The entries in the last column are neither all T nor all F.

$\therefore$ $(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$ is a contingency.

(Columns 5 and 6 : 1 mark; Column 7 : 1 mark; Conclusion : 1 mark)

(ii) The given equations can be written in matrix form as :

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$ (1 mark)

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

By $R_3 - 7R_2$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$ (1 mark)

By equality of matrices,

$x + 2y + z = 8$ ... (1)

$-y - 3z = -5$ ... (2)

$16z = 16$ ... (3)

From (3), $z = 1$

Substituting $z = 1$ in (2), we get

$-y - 3 = -5, \therefore y = 2$

Substituting $y = 2$, $z = 1$ in (1), we get

$x + 4 + 1 = 8 \qquad \therefore x = 3$

Hence, $x = 3$, $y = 2$, $z = 1$ is the required solution. (1 mark)

(iii) $e^x + e^y = e^{x+y}$ ... (1)

Differentiating both sides w.r.t. x, we get

$$e^x + e^y \cdot \frac{dy}{dx} = e^{x+y} \cdot \frac{d}{dx}(x + y)$$

$$\therefore e^x + e^y \frac{dy}{dx} = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$ (1 mark)

$$\therefore e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y} \cdot \frac{dy}{dx}$$

$$\therefore (e^y - e^{x+y}) \frac{dy}{dx} = e^{x+y} - e^x$$

$$\therefore \frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$ (1 mark)

$$= \frac{e^x + e^y - e^x}{e^y - e^x - e^y} \quad \text{... [By (1)]}$$

$$= \frac{e^y}{-e^x} = -e^{y-x}$$ (1 mark)

Q. 2. (B)

(i) $E_c = (0.0003)I^2 + (0.075)I$

$MPC = \frac{dE_c}{dI} = \frac{d}{dI}[(0.0003)I^2 + (0.075)I]$

$= (0.0003)(2I) + (0.075)(1)$

$= (0.0006)I + 0.075$

When $I = 1000$, then

$MPC = (0.0006)(1000) + 0.075$

$= 0.6 + 0.075 = 0.675.$ (1 mark)

$\because$ $MPC + MPS = 1$

$\therefore$ $0.675 + MPS = 1$

$\therefore$ $MPS = 1 - 0.675 = 0.325$ (1 mark)

Now, $APC = \frac{E_c}{I} = \frac{(0.0003)I^2 + (0.075)I}{I}$

$= (0.0003)I + (0.075)$

When $I = 1000$, then

$APC = (0.0003)(1000) + 0.075$

$= 0.3 + 0.075 = 0.375$ (1 mark)

$\because$ $APC + APS = 1$

$\therefore$ $0.375 + APS = 1$

$\therefore$ $APS = 1 - 0.375 = 0.625$ (1 mark)

Hence, $MPC = 0.675,$

$MPS = 0.325$

$APC = 0.375,$

$APS = 0.625.$

(ii) Let $I = \int \frac{x}{(x-1)^2(x+2)}\,dx$

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

$\therefore\ x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Put $x-1=0$, i.e. $x=1$, we get

$1 = A(0)(3) + B(3) + C(0) \qquad \therefore\ B = \frac{1}{3}$

Put $x+2=0$, i.e. $x=-2$, we get

$-2 = A(-3)(0) + B(0) + C(9) \qquad \therefore\ C = -\frac{2}{9}$ (1 mark)

Put $x=-1$, we get,

$-1 = A(-2)(1) + B(1) + C(4)$

But $B = \frac{1}{3}$ and $C = -\frac{2}{9}$

$\therefore\ -1 = -2A + \frac{1}{3} - \frac{8}{9}$

$\therefore\ 2A = -\frac{5}{9} + 1 = \frac{4}{9} \qquad \therefore\ A = \frac{2}{9}$ (1 mark)

$\therefore\ \frac{x}{(x-1)^2(x+2)} = \frac{(2/9)}{x-1} + \frac{(1/3)}{(x-1)^2} + \frac{(-2/9)}{x+2}$

$\therefore\ I = \int \left[\frac{(2/9)}{x-1} + \frac{(1/3)}{(x-1)^2} + \frac{(-2/9)}{x+2}\right] dx$

$= \frac{2}{9}\int \frac{1}{x-1}\,dx + \frac{1}{3}\int (x-1)^{-2}\,dx - \frac{2}{9}\int \frac{1}{x+2}\,dx$ (1 mark)

$= \frac{2}{9}\log|x-1| + \frac{1}{3}\cdot\frac{(x-1)^{-1}}{-1} - \frac{2}{9}\log|x+2| + c$

$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + c.$ (1 mark)

(iii) $(x^2 - y^2)dx + 2xy\,dy = 0$

$\therefore\ 2xy\,dy = -(x^2 - y^2)\,dx = (y^2 - x^2)\,dx$

$\therefore\ \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ ... (1)

Put $y = vx$ $\therefore\ \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes, $v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x \cdot vx}$ (1 mark)

$\therefore\ v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\therefore\ x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{v^2 - 1 - 2v^2}{2v}$

$\therefore\ x\frac{dv}{dx} = \frac{-1 - v^2}{2v} = -\left(\frac{1 + v^2}{2v}\right)$

$\therefore\ \frac{2v}{1 + v^2}dv = -\frac{1}{x}dx$ (1 mark)

Integrating, we get

$\int \frac{2v}{1 + v^2}dv = -\int \frac{1}{x}dx$

$\therefore\ \log|1 + v^2| = -\log x + \log c$

... $\left[\because\ \frac{d}{dv}(1 + v^2) = 2v \text{ and } \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c\right]$ (1 mark)

$\therefore\ \log\left|1 + \frac{y^2}{x^2}\right| = -\log x + \log c$

$\therefore\ \log\left|\frac{x^2 + y^2}{x^2}\right| = \log\left|\frac{c}{x}\right|$

$\therefore\ \frac{x^2 + y^2}{x^2} = \frac{c}{x}$

$\therefore\ x^2 + y^2 = cx$

This is the general solution. (1 mark)

Q. 3. (A)

(i) Let p : He studies.

q : He will go to college.

Then the symbolic form of the given statement is $p \to q$.

Converse : $q \to p$ is the converse of $p \to q$.

i.e. If he will go to college, then he studies. (1 mark)

Inverse : $\sim p \to \sim q$ is the inverse of $p \to q$.

i.e. If he does not study, then he will not go to college. (1 mark)

Contrapositive : $\sim q \to \sim p$ is the contrapositive of $p \to q$.

i.e. If he will not go to college, then he does not study. (1 mark)

(ii) $x = at^2$, $y = 2at$

Differentiating x and y w.r.t. t, we get

$$\frac{dx}{dt} = \frac{d}{dt}(at^2) = a\frac{d}{dt}(t^2)$$

$$= a \times 2t = 2at \quad \text{... (1)}$$

and $\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a\frac{d}{dt}(t)$

$$= 2a \times 1 = 2a$$ (1 mark)

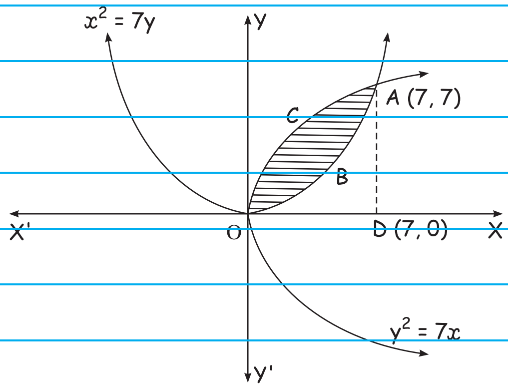
$$\therefore \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{2a}{2at} = \frac{1}{t}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{d}{dt}\left(\frac{1}{t}\right) \cdot \frac{dt}{dx}$$ (1 mark)

$$= -\frac{1}{t^2} \times \frac{1}{\left(\frac{dx}{dt}\right)} = -\frac{1}{t^2} \times \frac{1}{2at} \quad \text{... [By (1)]}$$

$$= -\frac{1}{2at^3}.$$ (1 mark)

(iii)



For finding the points of intersection of the two parabolas, we equate the values of y^2 from their equations.

From the equation $x^2 = 7y$, $y^2 = \frac{x^4}{49}$

$\therefore \frac{x^4}{49} = 7x \quad \therefore x^4 = 343x$

$\therefore x^4 - 343x = 0 \quad \therefore x(x^3 - 343) = 0$

$\therefore x = 0$ or $x^3 = 343$, i.e. $x = 7$

When $x = 0, y = 0$

When $x = 7, 7y = 49 \quad \therefore y = 7$

$\therefore$ the points of intersection are $O(0,0)$ and $A(7,7)$

Required area = area of the region OBACO

= (area of the region ODACO) −

(area of the region ODABO) (1 mark)

Now, area of the region ODACO

= area under the parabola $y^2 = 7x$, i.e. $y = \sqrt{7}\sqrt{x}$

$$= \int_0^7 \sqrt{7}\sqrt{x}\,dx = \sqrt{7}\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_0^7$$

$$= \sqrt{7} \times \frac{2}{3}\left[7^{\frac{3}{2}} - 0\right] = \frac{2\sqrt{7}}{3}[7\sqrt{7} - 0] = \frac{98}{3}$$ (1 mark)

Area of the region ODABO

= Area under the parabola $x^2 = 7y$ i.e. $y = \frac{x^2}{7}$

$$= \int_0^7 \frac{x^2}{7} dx = \frac{1}{7}\left[\frac{x^3}{3}\right]_0^7 = \frac{1}{7}\left[\frac{7^3}{3} - 0\right]$$

$$= \frac{7^2}{3} = \frac{49}{3}$$

$\therefore$ required area $= \frac{98}{3} - \frac{49}{3} = \frac{49}{3}$ sq units. (1 mark)

Q. 3. (B)

(i) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$

$$= 1(7-20) - 2(7-10) + 3(4-2)$$

$$= -13 + 6 + 6 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (1 mark)

By $R_2 - R_1$ and $R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $(-1) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$ (1 mark)

By $R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$ (1 mark)

By $R_1 - 7R_3$ and $R_2 + 2R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\therefore\ A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}.$$ (1 mark)

(ii) Let $I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx$ ... (1)

We use the property, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Hence in I, we replace x by $1+3-x$. (1 mark)

$$\therefore\ I = \int_1^3 \frac{\sqrt[3]{1+3-x+5}}{\sqrt[3]{1+3-x+5} + \sqrt[3]{9-1-3+x}}\,dx$$

$$= \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}}\,dx$$ ... (2) (1 mark)

Adding (1) and (2), we get

$$2I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx + \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x} + \sqrt[3]{x+5}}\,dx$$

$$= \int_1^3 \frac{\sqrt[3]{x+5} + \sqrt[3]{9-x}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx$$

$$= \int_1^3 1dx = [x]_1^3$$ (1 mark)

$$= 3 - 1 = 2$$

$$\therefore\ I = 1$$

Hence, $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\,dx = 1.$ (1 mark)

Q. 3. (C)

(i) Let x cm and y cm be the length and breadth of a rectangle.

Then its area is $xy = 50$.

$\therefore\ y = \dfrac{50}{\boxed{x}}$

Perimeter of the rectangle $= 2\ (x + y)$

$$= 2\left(x + \frac{50}{\boxed{x}}\right)$$ (1 mark)

Let $f(x) = 2\left(x + \dfrac{50}{\boxed{x}}\right)$

Then $f'(x) = 2\left(1 - \dfrac{50}{\boxed{x^2}}\right)$

and $f''(x) = 2\left(0 + \boxed{\dfrac{100}{x^3}}\right) = \dfrac{200}{\boxed{x^3}}$ (1 mark)

Now, $f'(x) = 0$, if $1 - \dfrac{50}{\boxed{x^2}} = 0$

i.e. if $x^2 = \boxed{50}$

i.e. if $x = \pm \boxed{5\sqrt{2}}$

But x is not negative.

$\therefore\ x = \boxed{5\sqrt{2}}$ and $f''\boxed{5\sqrt{2}} = \dfrac{200}{\boxed{(5\sqrt{2})^3}} > 0$ (1 mark)

$\therefore$ by the second derivative test, f is minimum at $x = \boxed{5\sqrt{2}}$

When $x = \boxed{5\sqrt{2}}$, $y = \dfrac{50}{\boxed{5\sqrt{2}}} = \boxed{5\sqrt{2}}$

$\therefore\ x = \boxed{5\sqrt{2}}$ cm, $y = \boxed{5\sqrt{2}}$ cm

Hence, the rectangle is a square of side $\boxed{5\sqrt{2}}$ cm. (1 mark)

(ii) $\frac{dy}{dx} + 2xy = x$ ... (1)

This is the linear differential equation of the form $\frac{dy}{dx} + P \cdot y = Q$,

where $P = \boxed{2x}$, $Q = \boxed{x}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \boxed{2x} dx} = e^{\boxed{x^2}}$ (1 mark)

$\therefore$ the solution of (1) is given by

$y \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.}) dx + c.$

$\therefore y \cdot \boxed{e^{x^2}} = \int \boxed{x} \cdot e^{\boxed{x^2}} dx + c$ ... (2) (1 mark)

Put $x^2 = t$

$\therefore 2x\, dx = dt$

$\therefore x\, dx = \frac{1}{2} dt$

$\therefore$ (2) becomes

$y \cdot \boxed{e^{x^2}} = \frac{1}{2} \int \boxed{e^t} dt + c$ (1 mark)

$\therefore y \cdot \boxed{e^{x^2}} = \frac{1}{2} \boxed{e^t} + c$

$\therefore y \cdot \boxed{e^{x^2}} = \frac{1}{2} e^{x^2} + c$ (1 mark)

This is the general solution.

SECTION - II

Q. 4. (A)

(i) (c) list price (1 mark)

(ii) (a) $\frac{\Sigma IW}{\Sigma W}$ (1 mark)

(iii) (a) 2 (1 mark)

(iv) (b) (2, 0) (1 mark)

(v) (b) DBCA (1 mark)

(vi) (b) $\frac{2}{9}$ (1 mark)

Q. 4. (B)

(i) False (1 mark)

(ii) False (1 mark)

(iii) True (1 mark)

Q. 4. (C)

(i) Annuitant (1 mark)

(ii) $y > x$ (1 mark)

(iii) counting (1 mark)

Q. 5. (A)

(i) $X =$ marks in Economics, $Y =$ Marks in mathematics.

We prepare the following table for calculation :

x	y	$(x-\bar{x})$ $\bar{x}=61$	$(y-\bar{y})$ $\bar{y}=80.4$	$(x-\bar{x})\cdot(y-\bar{y})$	$(x-\bar{x})^2$
59	78	-2	-2.4	4.8	4
60	82	-1	1.6	-1.6	1
61	82	0	1.6	0	0
62	79	1	-1.4	-1.4	1
63	81	2	0.6	1.2	4
$\Sigma x = 305$	$\Sigma y = 402$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(y-\bar{y}) = 0$	6.0 -3.0 $\Sigma(x-\bar{x})(y-\bar{y}) = 3$	$\Sigma(x-\bar{x})^2 = 10$

(1 mark)

Here, $n = 5$

$\bar{x} = \frac{\Sigma x}{n} = \frac{305}{5} = 61$; $\bar{y} = \frac{\Sigma y}{n} = \frac{402}{5} = 80.4$

Regression equation of Y on X :

$y = a + b_{yx}\cdot x$

$b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{3}{10} = 0.3$ (1 mark)

$a = \bar{y} - b_{yx}\cdot\bar{x}$

Putting $\bar{y} = 80.4$, $b_{yx} = 0.3$ and $\bar{x} = 61$, we get

$a = 80.4 - 0.3(61)$

$= 80.4 - 18.3 = 62.1$

$\therefore\ a = 62.1.$

Putting $a = 62.1$ and $b_{yx} = 0.3$ in $y = a + b_{yx}\cdot x$, we get the regression equation of Y on X as follows.

$y = 62.1 + 0.3x$

$\therefore\ y = 0.3x + 62.1.$ (1 mark)

(ii) We construct the following table to obtain 5-yearly moving averages for the given data :

Year t	Production x_t	5-yearly moving total	5-yearly moving averages Trend value
1974	0	-	-
1975	4	-	-
1976	9	30	6.0
1977	9	35	7.0
1978	8	35	7.0
1979	5	34	6.8
1980	4	35	7.0
1981	8	-	-
1982	10	-	-

(Column 2 : 1 mark; Column 3 : 1 mark; Column 4 : 1 mark)

(iii) Here, base year = 1995

$\therefore$ p_0 = Price in 1995 and

p_1 = Price in 2003.

Given : $P_{01} = 120$, $y = ?$

Commodity	Price (in ₹)	
	p_0	p_1
A	95	116
B	y	74
C	80	92
D	35	42
TOTAL	$\Sigma p_0 = 210 + y$	$\Sigma p_1 = 324$

(1 mark)

Now, Price Index Number

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

$$\therefore\ 120 = \frac{324}{210 + y} \times 100$$ (1 mark)

$$\therefore\ 120(210 + y) = 32400$$

$$\therefore\ 210 + y = \frac{32400}{120}$$

$$\therefore\ 210 + y = 270$$

$$\therefore\ y = 270 - 210$$

$$\therefore\ y = 60$$

Hence, the value of y is ₹ 60. (1 mark)

Q. (5) (B)

(i) Here Min. $(A) = 12$, Min. $(C) = 8$ and Max. $(B) = 12$.

Since, Min. $(A) \geqslant$ Max. (B) is satisfied, the problem can be converted into 5 types of toys. 2 machines problem and two fictitious machines are, $G = A + B$ and $H = B + C$

The problem now can be written as follows :

Types of toys	Processing time (in hours)	
	$G = A + B$	$H = B + C$
1	26	18
2	32	30
3	16	20
4	20	18
5	30	18

(1 mark)

Here, Min. $(G, H) = 16$, which corresponds to G.

Therefore, type 3 toy is processed at first.

3				

The problem now reduces to type 1, 2, 4, 5 toys.

Here, Min $(G, H) = 18$, which corresponds to H.

Therefore, type 1 toy is processed in the last, type 4 toy is processed at the last next to type 1 toy and type 5 toy is processed at last next to type 4 toy.

3		5	4	1

Now, type 2 toy is processed at last next to type 5 toy and the optimal sequence is obtained as follows :

3	2	5	4	1

Total elapsed time is obtained as follows :

Sequence of type of toy	Machine A		Machine B		Machine C		Idle time for Machine C
	Time in	Time out	Time in	Time out	Time in	Time out	
3	0	12	12	16	16	32	16
2	12	32	32	44	44	62	12
5	32	54	54	62	62	72	0
4	54	68	68	74	74	86	2
1	68	84	84	94	94	102	8
Total idle time for machine C							38

(2 marks)

Total elapsed time $T = 102$ hours

Idle time for machine A

$= T -$ Sum of processing time for all jobs on machine A

$= 102 - 84 = 18$ hours

Idle time for machine B

$= T -$ Sum of processing time for all jobs on machine B

$= 102 - 40 = 62$ hours

Idle time for machine $C = 38$ hours (1 mark)

(ii) Here, $n = 11$. We transform year t to u by taking $u = t - 1981$.

We construct the following table for calculation :

Year t	Production x_t	$u = t - 1981$	u^2	ux_t
1976	0	-5	25	0
1977	4	-4	16	-16
1978	4	-3	9	-12
1979	2	-2	4	-04
1980	6	-1	1	-06
1981	8	0	0	0
1982	5	1	1	5
1983	9	2	4	18
1984	4	3	9	12
1985	10	4	16	40
1986	10	5	25	50
Total	$\Sigma x_t = 62$	$\Sigma u = 0$	$\Sigma u^2 = 110$	125 $-$ 38 $\Sigma ux_t = 87$

(1 mark)

The equation of trend line is

$x_t = a' + b'u$

The normal equations are

$\Sigma x_t = na' + b'\Sigma u$ ... (1)

$\Sigma ux_t = a'\Sigma u + b'\Sigma u^2$ ... (2)

Here, $n = 11$, $\Sigma x_t = 62$, $\Sigma u = 0$, $\Sigma u^2 = 110$,

$\Sigma ux_t = 87$

Putting these values in normal equations, we get

$62 = 11a' + b'(0)$ ... (3)

$87 = a'(0) + b'(110)$ ... (4)

From equation (3), we get

$a' = \frac{62}{11} = 5.6364$

From equation (4), we get $b' = \frac{87}{110} = 0.7909$ (2 marks)

Putting $a' = 5.6364$ and $b' = 0.7909$ in the equation

$x_t = a' + b'u$, we get the equation of trend line as

$x_t = 5.6364 + 0.7909\,u$

Trend for the year 1990 : For $t = 1990$, $u = 1990 - 1981 = 9$

Putting $u = 9$ in $x_t = 5.6364 + 0.7909\,u$, we get

$x_{1990} = 5.6364 + 0.7909 \times 9$

$\therefore\ x_{1990} = 5.6364 + 7.1181 \qquad \therefore\ x_{1990} = 12.7545$

Hence, trend value for the year 1990 is 12.7545. (1 mark)

(iii) Let the company manufactures x units of chemical A and y units of chemical B. Then the total profit to the company is $p = ₹\,(350x + 400y)$. This is a linear function which is to be maximized. Hence, it is the objective function. (1 mark)

The constraints are as per the following table :

Raw Material ↓ \ Chemical →	A (x)	B (y)	Availability
P	3	2	120
Q	2	5	160

(1 mark)

The raw material P required for x units of chemical A and y units of chemical B is $3x + 2y$. Since, the maximum availability of P is 120, we have the first constraint as $3x + 2y \leqslant 120$.

Similarly, considering the raw material Q, we have $2x + 5y \leqslant 160$.

(1 mark)

Since, x and y cannot be negative, we have, $x \geqslant 0$, $y \geqslant 0$.

Hence, the given LPP can be formulated as :

Maximize $p = 350x + 400y$, subject to $3x + 2y \leqslant 120$,

$2x + 5y \leqslant 160$, $x \geqslant 0$, $y \geqslant 0$. (1 mark)

Q. 6. (A)

(i)

Commodity	Base Year		Current Year		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	p_0	q_0	p_1	q_1				
I	8	30	12	25	360	240	300	200
II	10	42	20	16	840	420	320	160
Total	-	-	-	-	$\Sigma p_1q_0 = 1200$	$\Sigma p_0q_0 = 660$	$\Sigma p_1q_1 = 620$	$\Sigma p_0q_1 = 360$

(1 mark)

Laspeyre's Price Index Number :

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$= \frac{1200}{660} \times 100$$

$$= 1.8181 \times 100$$

$$= 181.82.$$

(1 mark)

Paasche's Price Index Number :

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{620}{360} \times 100$$

$$= 1.7222 \times 100$$

$$= 172.22$$

(1 mark)

Hence, Laspeyre's and Paasche's Price Index Numbers are 181.82 and 172.22 respectively.

(ii) Since, it is a maximization problem, subtract each of the elements in the matrix from the largest element of the matrix which is 16 here.

Salesmen	Districts 1	2	3	4
A	0	6	4	5
B	4	3	1	1
C	1	1	5	2
D	3	2	2	1

(1 mark)

Step 1 : Subtract the minimum (smallest) element of each row from the elements of that row.

Salesmen	Districts 1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

Step 2 : Subtract the smallest element of each column from the elements of that column.

Salesmen	Districts 1	2	3	4
A	0	6	4	5
B	3	2	0	0
C	0	0	4	1
D	2	1	1	0

(1 mark)

Step 3 : Since, the number of lines covering zeros is 4 equal to the order of matrix 4. The optimal solution has reached.

Salesmen	Districts 1	2	3	4
A	[0]	6	4	5
B	3	2	[0]	~~0~~
C	~~0~~	[0]	4	1
D	2	1	1	[0]

(1 mark)

The following optimal solution is obtained :

Salesmen	Districts	Profit (₹)
A	1	16
B	3	15
C	2	15
D	4	15

Total profit = ₹ 61 (1 mark)

(iii) 2 fair coins are tossed. $\therefore$ $S = \{HH, HT, TH, TT\}$

Let X = number of heads $\therefore$ $X = \{0, 1, 2\}$

Now, $P(X=0) = \frac{1}{4}$, $P(X=1) = \frac{2}{4}$, $P(X=2) = \frac{1}{4}$ (1 mark)

Let x_i = the amount received corresponds to the values of X.

We construct the following table to compute the expected winning amount and the variance of winning amount :

X	x_i ₹	$P(X=x)$ p_i	$x_i\ p_i$	$x_i^2\ p_i = x_i p_i \times x_i$
0	2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
1	5	$\frac{2}{4}$	$\frac{10}{4}$	$\frac{50}{4}$
2	10	$\frac{1}{4}$	$\frac{10}{4}$	$\frac{100}{4}$
Total	-	1	$\Sigma x_i p_i = \frac{22}{4}$	$\Sigma x_i^2 p_i = \frac{154}{4}$

(1 mark)

Expected winning amount :

$E(X) = \Sigma x_i\ p_i = \frac{22}{4} =$ ₹ 5.5

Variance of winning amount :

$Var(X) = \Sigma x_i^2\ p_i - (\Sigma x_i\ p_i)^2$

$= \frac{154}{4} - (5.5)^2 = 38.5 - 30.25$

$= 8.25 =$ ₹ 8.25 (1 mark)

Q. 6. (B)

(i) Given : $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.2$, $b_{xy} = -0.3$

(a) Correlation coefficient between X and Y :

$r = \pm\sqrt{b_{yx} \cdot b_{xy}}$ (1 mark)

$= \pm\sqrt{(-1.2)(-0.3)}$

$= \pm\sqrt{0.36}$

$\therefore\ r = -0.6$ ... ($\because$ b_{yx} and b_{xy} are negative.) (1 mark)

(b) Estimation of Y for X = 50 :

Regression equation of Y on X is,

$y = a + b_{yx} \cdot x$

$b_{yx} = -1.2$

$a = \bar{y} - b_{yx} \cdot \bar{x}$

$= 28 - (-1.2)53$

$= 28 + 63.6 = 91.6$

$\therefore\ y = 91.6 - 1.2x$

$\therefore\ y = -1.2x + 91.6$

Put $x = 50$ $\therefore\ y = -1.2\,(50) + 91.6$

$\therefore\ y = -60 + 91.6$ $\therefore\ y = 31.6$

Hence, $y = 31.6$, when $x = 50$ (1 mark)

(c) Estimation of X for Y = 25 :

Regression equation of X on Y is,

$x = a' + b_{xy} \cdot y$ $b_{xy} = -0.3$

$a' = \bar{x} - b_{xy}(\bar{y})$

$= 53 - (-0.3)(28)$

$= 53 + 8.4 = 61.4$

$\therefore\ x = 61.4 - 0.3y$

$\Rightarrow x = -0.3y + 61.4$

Put $y = 25$, $\therefore\ x = -0.3\,(25) + 61.4$

$\therefore\ x = -7.5 + 61.4$ $\therefore\ x = 53.9$

Hence, $x = 53.9$, when $y = 25$ (1 mark)

(ii) Here, X = Number of complaints a bank manager receives per day

$X \sim P(m = 4)$, $e^{-4} = 0.0183$

$\therefore\ P[X = x] = \dfrac{e^{-m}m^{x}}{x!}$

$= \dfrac{e^{-4}(4)^{x}}{x!}$

$= 0.0183 \times \dfrac{(4)^{x}}{x!}$ (1 mark)

(a) P[Only two complaints on a given day]

$= P[X = 2]$

$= 0.0183 \times \dfrac{(4)^{2}}{2!}$

$= 0.0183 \times \dfrac{16}{2}$

$= 0.0183 \times 8$

$= 0.1464$

Hence, probability that only two complaints on a given day is 0.1464.

(1 mark)

(b) P[At most two complaints on a given day]

$= P[X \leqslant 2]$

$= P[X = 0] + P[X = 1] + P[X = 2]$

$= 0.0183 \times \dfrac{4^{0}}{0!} + 0.0183 \times \dfrac{(4)^{1}}{1!} + 0.0183\,\dfrac{(4)^{2}}{2!}$ (1 mark)

$= 0.0183\,(1 + 4 + 8)$

$= 0.0183 \times 13 = 0.2379$

Hence, probability that at most two complaints on a given day is 0.2379. (1 mark)

Q. 6. (C)

(i) Face value (SD) = ₹ 7000, r = 5%, Cash value = ₹ 6930,

BD = SD − CV = ₹ 7000 − ₹ 6930 = ₹ 70. (1 mark)

Date of drawing the bill = 14th April 2019

Date of discounting the bill = 6th July 2019.

$$BD = \frac{SD \times n \times r}{100}$$

$$\therefore\ 70 = 7000 \times \frac{n}{100} \times 5$$

$$\therefore\ n = \frac{70}{350} \text{ years}$$

$$\therefore\ n = \frac{70}{350} \times 365 = 73 \text{ days.}$$ (1 mark)

For legal due date, 73 days are counted from due date of discounting the bill i.e. from 6th July 2019.

July 2019	Aug. 2019	Sept. 2019	Total
25	31	17	73

Legal due date : 17th Sept. 2019. (1 mark)

Nominal due date is 14th Sept. 2019

Period of the bill is 14th Sept. 2019 − 14th April 2019 = 5 months.

(1 mark)

(ii) Present value P = ₹ 10000

Accumulated value A = ₹ 20,000 $\quad r = 12\%$

$i = \frac{r}{100} = \frac{12}{100} = 0.12$ (1 mark)

C = Payment of each annuity

$\frac{1}{P} - \frac{1}{A} = \frac{i}{C}$ (1 mark)

$\therefore\ 0.0001 - \frac{1}{20000} = \frac{0.12}{C}$ (1 mark)

$\therefore\ 0.0001 - 0.00005 = \frac{0.12}{C}$

$\therefore\ C = \frac{0.12}{0.00005}$ (1 mark)

$\therefore$ C = ₹ 2400.